

MODULE-3: DIGITAL ELECTRONICS

Syllabus: Introduction, Switching and logic levels, Digital Waveform, Number System: Decimal number system, Binary number system, Converting Decimal to Binary, Hexadecimal Number System, Converting Binary to Hexadecimal, Hexadecimal to Binary, Converting Hexadecimal to Decimal, Converting Decimal to Hexadecimal, Octal numbers: Binary to octal conversion, Complement of Binary Numbers, Boolean algebra theorem, DeMorgan's theorem, Digital Circuits: logic gates, NOT gate, AND gate, OR gate, XOR gate, NAND gate, NOR gate, X-NOR gate, Algebraic Simplification, NAND & NOR implementation: NAND implementation, NOR implementation, Half adder, Full adder.

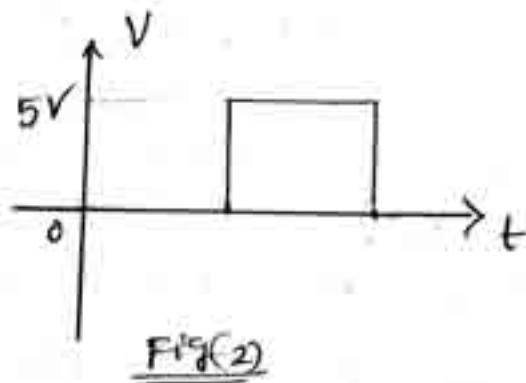
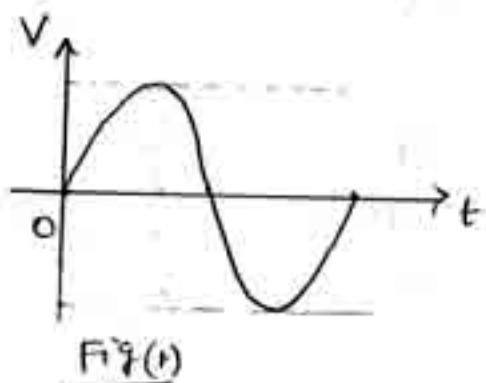
* Introduction:

→ A continuously varying signal (Voltage or current) is called an analog signal

Ex: Sinusoidal Voltage (fig 1)

→ A Signal (Voltage or current) which is having only two discrete values is called a digital signal

Ex: Square wave (fig 2)



Analog domain	Digital domain
① An analog signal can have infinite number of values	① A digital signal can have only two values (5V or 0V) These values are, high(ON) & low(OFF)(0)
② Analog systems are generally difficult to design	② Digital systems are generally easier to design.
③ Information storage is difficult	③ Information storage is easy
④ Accuracy & precision are less	④ Accuracy & precision are greater
⑤ Operation cannot be programmed	⑤ Operation can be programmed
⑥ Analog circuits are affected by noise	⑥ Digital circuits are less affected by noise
⑦ Analog signals consume less bandwidth(BW)	⑦ Digital signals consume more BW.
⑧ Analog circuit are controlled using R,L,C & op-amps etc	⑧ Digital circuit are controlled using adder, multiplier, memory etc

* Switching and Logic level:

The switching circuit is shown in

Fig ③.

When the switch 'S' is open @ OFF @ False @ NO @ Low, the output voltage is $V_o = 5V$ @ ON @ High @ True @ Yes.

When the switch 'S' is closed @ ON @ True @ Yes, the output $V_o = 0V$ @ Low @ OFF @ False.

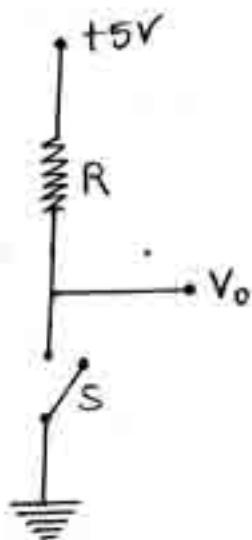


Fig ③ : Switching Circuit

Therefore input '0' results in output '1' & vice versa i.e. hence this circuit is called as switching circuit @ State inversion circuit @ Inverter
 ④ NOT gate.

NOT gate symbol is shown in fig ④

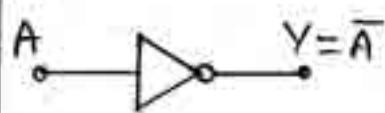


Fig ④

A	Y
0	1
1	0

Fig ⑤

Fig ⑤ shows the truth table (input-output relationship) of NOT gate

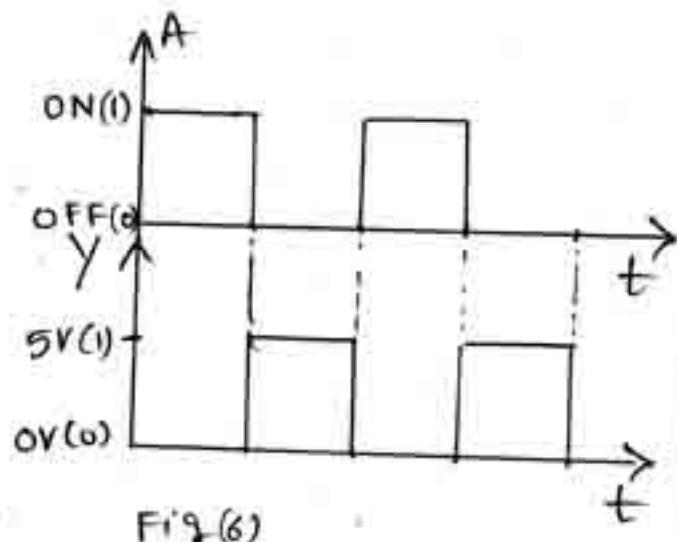


Fig ⑥

Fig ⑥ shows the input-output waveforms of NOT gate.

Note:

(O, 9, 1x)

- ① The branch of electronics which deals with digital circuits is called digital electronics.
- ② An electronic circuit that handles only a digital signal (Low @ high) is called a digital circuit.
- ③ Buffer: (Noninverter)



Buffer symbol

A	Y
0	0
1	1

Buffer truth table

- ④ Types of Number System (digital number system)
 - a) Decimal Ex: $(15)_{10}$ @ 15_{10} (base 0 to 9)

- ⑥ Binary EX: $(101)_2 @ 101_2$ (Value 0 and 1)
 ⑦ Octal EX: $(37)_8 @ 37_8$ (Value 0 to 7)
 ⑧ Hexadecimal EX: $(3AC)_{16} @ 3AC_{16}$ (Value 0 to 9 &
 etc. A, B, C, D, E, F)

- ⑨ Each binary digit (0@1) is bit
 A string of four bits is nibble (1010)
 A string of eight bits is byte. ($1010\ 0110$)
 ⑩ Digital circuits which accomplish some operation
 (Boolean algebra) are logic gates
 ⑪ Branch of algebra which deals with only 0's & 1's
 is called Boolean algebra. (Algebra used to symbolically
 describe logic functions)

* Digital Waveform:

Ideally:

$$\text{High} = 5V = 1$$

$$\text{Low} = 0V = 0$$

Practically:

Voltages at different points
 may slightly vary (due to
 internal resistances, parasitic
 effects & loading effects)

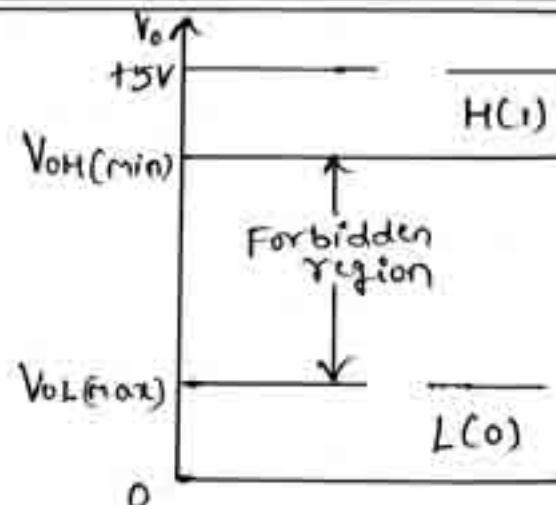


Fig. 7: Voltage ranges for logic levels

∴ High Voltage = Voltage between $+5V$ & $V_{OH(\min)}$
 (Ex: 5V to 3.5V)

Low Voltage = Voltage between $0V$ & $V_{OL(\max)}$
 (Ex: 0V to 0.2V)

Forbidden region = $V_{OH(\min)} - V_{OL(\max)}$ (Ex: $3.5 - 0.2 = 3.3V$)
 [Neither H(1) or L(0)]

* Number System :

There are four number systems that are used in digital system:

- 1). Decimal
- 2). Binary
- 3). Octal
- 4) Hexadecimal.

①. Decimal number system

→ Decimal number system uses the digits 0 to 9 (10)

→ Any number N in a Positional number system (2) radix-weighted positional number system is represented by,

$$N = d_{n-1} \gamma^{n-1} + d_{n-2} \gamma^{n-2} + \dots + d_1 \gamma^1 + d_0 \gamma^0 \quad \text{--- (1)}$$

$$= d_{n-1} \times \gamma^{n-1} + d_{n-2} \times \gamma^{n-2} + \dots + d_1 \times \gamma^1 + d_0 \times \gamma^0 \quad \text{--- (2)}$$

where, $d_i \rightarrow$ digit in the number system $[0 \leq d_i \leq (\gamma-1)]$

$\gamma \rightarrow$ base or radix of the number system.

$n \rightarrow$ number of digits in the integer part of N .

$m \rightarrow$ number of digits in the fraction part of N .

Ex: $2438 = 2000 + 400 + 30 + 8$

$$= 2 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$$

$$N = 2438, d_{n-1} = 2, d_{n-2} = 4, \dots, d_0 = 8, \gamma = 10, n = 4$$

②. Binary number system

→ Binary number system uses the digits 0 & 1 (2)

→ Ex: $1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \quad \text{--- (3)}$

$$= 8 + 4 + 0 + 1$$

$$= 13_{10}$$

Comparing (2) & (3), we can write $d_3 = 1, d_2 = 1, d_1 = 0, d_0 = 1$

$$N = 1101, \gamma = 2, n = 4.$$

③ Octal number System

→ Octal number system uses the digits 0 to 7 (8)

$$\rightarrow \text{Ex: } 4367_2 = 4 \times 8^4 + 3 \times 8^3 + 6 \times 8^2 + 7 \times 8^1 + 2 \times 8^0 - ④$$

$$= 16384 + 1536 + 384 + 56 + 2$$

$$= 18362_{10}$$

Comparing ② & ④, we can write,

$$d_4 = 4, d_3 = 3, d_2 = 6, d_1 = 7, d_0 = 2, n = 5, \gamma = 8$$

$$N = 4367_2$$

④ Hexadecimal number System

→ Hexadecimal number system uses the digits 0 to 9

& A, B, C, D, E, F (16)

$$\rightarrow \text{Ex: } AB3 = 10 \times 16^2 + 11 \times 16^1 + 3 \times 16^0 - ⑤$$

$$= 2560 + 176 + 3$$

$$= 2739_{10}$$

Comparing ② & ⑤, we can write

$$d_2 = A = 10, d_1 = B = 11, d_0 = 3, n = 3, \gamma = 16, N = AB3$$

* Conventions :

① Decimal to Binary:

② Convert the decimal number 109 to binary

Ans:

2	109
2	54 - 1
2	27 - 0
2	13 - 1
2	6 - 1
2	3 - 0
1	1 - 1

$$\therefore (109)_{10} = (1101101)_2$$

(b) Perform the following:

$$(i) (69)_{10} = (?)_2 \quad (ii) 53.75_{10} = ?_2 \quad (iii) 0.375_{10} = ?_2$$

Ans:

$$\begin{array}{r} 2 | 69 \\ 2 | 34 -1 \\ 2 | 17 -0 \\ 2 | 8 -1 \\ 2 | 4 -0 \\ 2 | 2 -0 \\ 1 -0 \end{array}$$

$$\therefore (69)_{10} = (1000101)_2$$

(ii) Consider Whole Integer Part

$$\begin{array}{r} 2 | 53 \\ 2 | 26 -1 \\ 2 | 13 -0 \\ 2 | 6 -1 \\ 2 | 3 -0 \\ 1 -1 \end{array}$$

$$\therefore (53)_{10} = (110101)_2$$

Consider fractional Part

$$\begin{array}{l} 0.75 \times 2 = 1.5 \quad 1 \\ 0.5 \times 2 = 1 \quad 1 \\ \vdots \end{array}$$

$$\therefore 0.75_{10} = 0.11_2$$

$$\therefore (53.75)_{10} = (110101.11)_2 //$$

(iii)

Integer

$$\begin{array}{ll} 0.375 \times 2 = 0.75 & 0 \\ 0.75 \times 2 = 1.5 & 1 \\ 0.5 \times 2 = 1 & 1 \end{array}$$

$$\therefore 0.375_{10} = 0.011_2 //$$

2) Binary to Decimal:

a) Convert the following binary numbers to decimal numbers.

$$(i) 1101 \quad (ii) 10001.11 \quad (iii) 0.1011$$

Ans: (i) $1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 4 + 1 = (13)_{10}$

(ii) $10001.11 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^{-2} = 16 + 8 + 2 + 0.25 = (26.25)_{10} //$

(iii) $0.1011 = 1 \times 2^{-1} + 1 \times 2^{-3} + 1 \times 2^{-4} = 0.5 + 0.125 + 0.0625 = (0.6875)_{10} //$

Q) Perform the following (i) $(0.1011)_2 = (?)_{10}$

Ans:

$$\begin{aligned}0.1011 &= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\&= 0.5 + 0 + 0.125 + 0.0625\end{aligned}$$

$$(0.1011)_2 = (0.6875)_{10} //.$$

Q) Decimal to octal:

Q) Convert the following decimal numbers to octal numbers

- (i) 359 (iii) 658.825 (ii) 0.245 (iv) 254

Ans:

(i) $8 \overline{) 359}$
 $\underline{8 \mid 44 - 7}$
 $\underline{\underline{5 - 4}}$

$$\therefore (359)_{10} = (547)_8 //$$

(ii) Consider integer Part (Whole no)

$$\begin{array}{r} 8 \mid 658 \\ 8 \mid 82 - 2 \\ 8 \mid 10 - 2 \\ \underline{\underline{1 - 2}} \end{array}$$

Consider fractional Part

$$\begin{array}{l} 0.825 \times 8 = 6.6 \quad 6 \\ 0.6 \times 8 = 4.8 \quad 4 \\ 0.8 \times 8 = 6.4 \quad 6 \\ 0.4 \times 8 = 3.2 \quad 3 \end{array}$$

$$\therefore (658)_{10} = (1222)_8, \therefore 0.825_{10} = 0.6463_8$$

$$\therefore (658.825)_{10} = (1222.6463)_8 //$$

(iii) .

$$0.245 \times 8 = 1.96$$

Integer

$$\underline{1}$$

$$0.96 \times 8 = 7.68$$

$$\underline{7}$$

$$0.68 \times 8 = 5.44$$

$$\underline{5}$$

$$0.44 \times 8 = 3.52$$

$$\underline{3}$$

$$\therefore (0.245)_{10} = (0.1753)_8 //$$

(iv)

$$\begin{array}{r} 8 \mid 254 \\ 8 \mid 31 - 6 \\ \underline{\underline{3 - 7}} \end{array}$$

$$(254)_{10} = (376)_8 //$$

Q) Perform the following upto 2 octal places

$$(0.44)_{10} = ?_8$$

Sol:

Integer

$$0.44 \times 8 = 3.52 \quad 3 \\ 0.52 \times 8 = 4.16 \quad 4 \downarrow$$

~~0.44~~ $\therefore (0.44)_{10} = 0.34_8$

4) Octal to Decimal:

a) Perform the following

(i) $(133)_8 = (?)_{10}$ (ii) $372_8 = ?_{10}$ (iii) $24.6_8 = ?_{10}$

(iv) $0.76_8 = ?_{10}$

Sol:

(i) $133_8 = 1 \times 8^2 + 3 \times 8^1 + 3 \times 8^0 = 64 + 24 + 3 = 91_{10}$

(ii) $372_8 = 3 \times 8^2 + 7 \times 8^1 + 2 \times 8^0 = 192 + 56 + 2 = 250_{10}$

(iii) $24.6_8 = 2 \times 8^1 + 4 \times 8^0 + 6 \times 8^{-1} = 16 + 4 + 0.75 = 20.75_{10}$

(iv) $0.76_8 = 7 \times 8^{-1} + 6 \times 8^{-2} = 0.875 + 0.09375 = 0.96875_{10}$

b) Convert the following octal numbers to decimal numbers

(i) 564 (iii) 234.56

Sol:

(i) $564_8 = 5 \times 8^2 + 6 \times 8^1 + 4 \times 8^0 = 320 + 48 + 4 = 372_{10}$

(iii) $234.56_8 = 2 \times 8^2 + 3 \times 8^1 + 4 \times 8^0 + 5 \times 8^{-1} + 6 \times 8^{-2}$
 $= 128 + 24 + 4 + 0.625 + 0.09375$

$= 156.71875_{10}$

5) Decimal to Hexadecimal

a) Perform the following

(i) $(541)_{10} = (?)_{16}$ (ii) $378_{10} = ?_{16}$

Ques:

$$\begin{array}{r} 16 \Big| 541 \\ 16 \Big| 33 - 13(D) \\ 2 - 1 \end{array}$$

$$\begin{array}{r} 16 \Big| 378 \\ 16 \Big| 23 - 10(A) \\ 1 - 7 \end{array}$$

$$\therefore \boxed{(541)_{10} = (21D)_{16}}$$

$$\therefore \boxed{378_{10} = 17A_{16}}$$

~~.....~~

Ques: Convert the following decimal number to Hexadecimal number.

(i) 5386.345 (ii) 0.256 up to 3 Hexadecimal places

Ques:

(i) Consider integer part
(Whole number)

$$\begin{array}{r} 16 \Big| 5386 \\ 16 \Big| 336 - 10(A) \\ 16 \Big| 21 - 0 \\ 1 - 5 \end{array}$$

Consider fractional part

Integer

$$0.345 \times 16 = 5.52$$

5

$$0.52 \times 16 = 8.32$$

8

$$0.32 \times 16 = 5.12$$

5

$$0.12 \times 16 = 1.92$$

1

$$\therefore 5386_{10} = 150A_{16}$$

$$\therefore 0.345_{10} = 0.5851_{16}$$

$$\therefore \boxed{(5386.345)_{10} = (150A.585)_{16}} //$$

(ii)

Integer

$$0.256 \times 16 = 4.096 \quad 4$$

$$0.096 \times 16 = 1.536 \quad 1$$

$$0.536 \times 16 = 8.576 \quad 8$$

 \downarrow

$$\therefore \boxed{0.256_{10} = 0.418_{16}} //$$

⑥ Hexadecimal to decimal

⑦ Perform the following

(i) $(FACE)_{16} = (?)_{10}$ (ii) $(AB.32)_{16} = (?)_{10}$

Sol:

$$\begin{aligned}(i) (FACE)_{16} &= F \times 16^3 + A \times 16^2 + C \times 16^1 + E \times 16^0 \\ &= 15 \times 16^3 + 10 \times 16^2 + 12 \times 16^1 + 14 \times 1^0 \\ &= 61440 + 2560 + 192 + 14\end{aligned}$$

$$(FACE)_{16} = (64206)_{10} //.$$

$$\begin{aligned}(ii) (AB.32)_{16} &= A \times 16^1 + B \times 16^0 + 3 \times 16^{-1} + 2 \times 16^{-2} \\ &= 10 \times 16 + 11 \times 1 + \frac{3}{16} + \frac{2}{16^2} \\ &= 160 + 11 + 0.1875 + 0.0078125\end{aligned}$$

$$(AB.32)_{16} = (171.1953125)_{10}$$

⑧ Convert the following hexadecimal number to decimal number.

(i) $3ABH.12$ (ii) $ABCDE$

$$\begin{aligned}\text{Sol: } 3ABH.12 &= 3 \times 16^3 + A \times 16^2 + B \times 16^1 + 1 \times 16^0 + 2 \times 16^{-2} \\ &= 3 \times 16^3 + 10 \times 16^2 + 11 \times 16^1 + 1 \times 1 + \frac{1}{16} + \frac{2}{16^2} \\ &= 12288 + 2560 + 176 + 1 + 0.0625 + 0.0078125\end{aligned}$$

$$3ABH.12_{16} = 15028.07031_{10} //$$

$$\begin{aligned}(iii) ABCDE_{16} &= A \times 16^4 + B \times 16^3 + C \times 16^2 + D \times 16^1 + E \times 16^0 \\ &= 10 \times 16^4 + 11 \times 16^3 + 12 \times 16^2 + 13 \times 16^1 + 14 \times 1^0 \\ &= 655360 + 45056 + 3072 + 208 + 14\end{aligned}$$

$$ABCDE_{16} = 703710_{10} //$$

Note:

- ① What is the largest number that can be represented using eight bits?

Ans: $N = 8$

$$\text{Largest number} = 2^N - 1 = 2^8 - 1 = 255_{10} = \underline{\underline{11111111}}_2$$

- ② Determine the value of base x , if

(i) $(225)_x = (341)_8$ (ii) $(211)_x = (152)_8$

Ans:

(i) Given

$$(225)_x = (341)_8 \quad \text{--- (1)}$$

Convert $(341)_8$ to decimal

$$(341)_8 = 3 \times 8^2 + 4 \times 8^1 + 1 \times 8^0 \\ = 192 + 32 + 1$$

$$(341)_8 = (225)_{10} \quad \text{--- (2)}$$

From (1) & (2), we get

$$\boxed{x = 10}$$

(ii) Given

$$(211)_x = (152)_8$$

Convert $(152)_8$ to decimal

$$(152)_8 = 1 \times 8^2 + 5 \times 8^1 + 2 \times 8^0 \\ = 64 + 40 + 2$$

$$(152)_8 = (106)_{10} \quad \text{--- (1)}$$

$$(211)_x = 2 \times x^2 + 1 \times x + 1 \times x^0$$

$$(211)_x = 2x^2 + x + 1 \quad \text{--- (2)}$$

From (1) & (2), we get

$$2x^2 + x + 1 = 106$$

$$\Rightarrow 2x^2 + x - 105 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{cases} a = 2 \\ b = 1 \\ c = -105 \end{cases}$$

$$= \frac{-1 \pm \sqrt{1^2 + 4 \times 2 \times 105}}{2 \times 2}$$

$$= \frac{-1 \pm 29}{4}$$

$$= 7 \quad \text{or} \quad -7.5$$

$$\therefore \boxed{x = 7} //$$

③ Octal - binary numbers

Octal number	Binary Equivalent
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Table(1)

④ To write any number

in binary we use

$$\begin{array}{cccccc}
 & 4 & 3 & 2 & 1 & 0 \\
 2 & 2 & 2 & 2 & 2 & 2 \\
 (10) & (8) & (4) & (2) & (1) & (Using)
 \end{array}$$

 (10) (8) (4) (2) (1) (5 bits)

Ex:

$$10 = 8 + 2 \rightarrow 01010$$

$$14 = 8 + 4 + 2 \rightarrow 01110$$

⑤ Hexadecimal - binary - decimal

Decimal number	Hexadecimal number	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Table(2)

Extra numerical:

i) convert octal 143 to binary

Ans:

$$143_8 = (001\ 100\ 011)_2$$

[From table(1)]

ii) Perform the following

$$(i) (232.76)_8 = (?)_2 \quad (ii) (43.26)_8 = ?_2$$

Ans: (i) $232.76_8 = (010\ 011\ 010 \cdot 111\ 10)_2 //$

$$(ii) 43.26_8 = (100\ 011. 010\ 110)_2 //$$

5) Perform the following

$$(i) (110101)_2 = (?)_8 \quad (ii) (10100)_2 = (?)_8$$

$$(iii) (1101001. 1101)_2 = ?_8$$

Rul:

$$(i) \begin{array}{r} 6 \\ | \\ 110 \end{array} \begin{array}{r} 5 \\ | \\ 101 \end{array} \therefore (10101)_2 = (65)_8 //$$

$$(ii) (10100)_2 = \underbrace{010}_2 \underbrace{100}_4 = \underline{\underline{(24)}_8}$$

$$(iii) 1101001. 1101 = \underbrace{001}_1 \underbrace{101}_2 \underbrace{001}_1. \underbrace{110}_2 \underbrace{100}_4 = \underline{\underline{(151.64)}_8}$$

6) Perform the following

$$(i) FAFB_{16} = ?_2 \quad (ii) (A3.D24)_{16} = (?)_2$$

Rul:

$$(i) FAFB_{16} = \underbrace{1111}_F(15) \underbrace{1010}_A(10) \underbrace{1111}_F(15) \underbrace{1011}_B(11)$$

$$\therefore FAFB_{16} = 1111101011111011_2 //$$

$$(iii) (A3.D24)_{16} = \underbrace{1010}_A(10) \underbrace{0011}_3 \cdot \underbrace{1101}_D(13) \underbrace{0010}_2 \underbrace{0100}_4$$

$$\therefore (A3.D24)_{16} = (10100011. 110100100100)_2 //$$

7) Perform the following

$$(i) (4567)_{10} = (?)_2 = (?)_8 = (?)_{16}$$

Rul: (ii) $(101101110. 11011)_2 = (?)_{16}$

$$\begin{array}{r}
 2 | 4567 \\
 2 | 2283 - 1 \\
 2 | 1141 - 1 \\
 2 | 570 - 1 \\
 2 | 285 - 0 \\
 2 | 142 - 1 \\
 2 | 71 - 0 \\
 2 | 35 - 1 \\
 2 | 17 - 1 \\
 2 | 8 - 1 \\
 2 | 4 - 0 \\
 2 | 2 - 0 \\
 1 - 0
 \end{array}$$

$$\begin{array}{r}
 0010000111010111 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1 \quad 0 \quad 1 \quad 2 \quad 7
 \end{array}$$

$$\therefore (1000111010111)_2 = \underline{\underline{10727}}_8$$

$$\begin{array}{r}
 0001000111010111 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1 \quad 1 \quad D \quad 7
 \end{array}$$

$$\therefore (1000111010111)_2 = (11D7)_{16}$$

$$(4567)_{10} = (100011\underline{1}010111)_2$$

$$\boxed{(4567)_{10} = (1000111010111)_2 = (10727)_8 = (11D7)_{16}}$$

(iii)

$$\begin{array}{r}
 000101101110.11011000 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1 \quad 6 \quad E \cdot D \quad 8
 \end{array}$$

$$\boxed{(10110110.11011)_2 = (16E \cdot D8)_{16}}$$

* Binary addition: [0 & 1]

Rules

$$\begin{array}{r}
 \textcircled{1} \quad 0 \\
 + 0 \\
 \hline
 0 \quad 0 \\
 \uparrow \quad \uparrow \\
 \text{carry} \quad \text{sum}
 \end{array}$$

$$\begin{array}{r}
 \textcircled{2} \quad 0 \\
 + 1 \\
 \hline
 ? \quad 1 \\
 \uparrow \quad \uparrow \\
 \text{carry} \quad \text{sum}
 \end{array}$$

$$\begin{array}{r}
 \textcircled{3} \quad 1 \\
 + 0 \\
 \hline
 ? \quad 1 \\
 \uparrow \quad \uparrow \\
 \text{carry} \quad \text{sum}
 \end{array}$$

$$\begin{array}{r}
 \textcircled{4} \quad 1 \\
 + 1 \\
 \hline
 ? \quad 0 \\
 \uparrow \quad \uparrow \\
 \text{carry} \quad \text{sum}
 \end{array}$$

$$\begin{array}{r}
 + 1 \\
 \hline
 2 \quad 1 \\
 \uparrow \quad \uparrow \\
 \text{carry} \quad \text{sum}
 \end{array}$$

$$\begin{array}{r} 5 \\ \underline{+ 1} \\ 1 \end{array}$$

↑ ↑

carry sum

$$\begin{array}{r}
 & 1 \\
 & | \\
 & 1 \\
 + & 1 \\
 \hline
 3
 \end{array}
 \quad
 \begin{array}{r}
 2 \boxed{3} \\
 | - | \\
 \text{carry} \quad \text{sum}
 \end{array}$$

(i) Add the following binary numbers

④ 11011 & 10111 ⑤ 101 & 011 ⑥ 11.01 & 10.11

$$\begin{array}{r} \textcircled{a} \quad \begin{array}{r} 1101 \\ + 1011 \\ \hline (140010)_2 \end{array} & \textcircled{b} \quad \begin{array}{r} 101 \\ + 011 \\ \hline (\underline{\underline{1000}})_2 \end{array} & \textcircled{c} \quad \begin{array}{r} 11.01 \\ + 10.11 \\ \hline (\underline{\underline{110.00}})_2 // \end{array} \end{array}$$

* Octal & hexadecimal addition: [(0-7 for Octal) & (0-9 & A-F for Hexa)]

④ Perform the following

$$(ii) \begin{array}{r} (13.26)_8 \\ + (31.42)_8 \\ \hline (?)_8 \end{array} \quad (iii) \begin{array}{r} (ABCD)_{16} \\ + (AB32)_{16} \\ \hline (?)_{16} \end{array}$$

$$\begin{array}{r} \text{Rels} \\ \hline (\text{F}) \quad (43.2\%) \\ + (31.4\%) \\ \hline (74.7\%) \end{array}$$

$$\begin{array}{r} & 1 \\ & (A B C D)_{16} \\ \frac{6}{8} \frac{2}{8} 8 & \underline{18} \\ 1 & + (A B 3 2)_{16} \\ \hline (156FF)_{16} \end{array}$$

$$\begin{array}{r} B \\ B \\ \hline 22 \\ 16 \overline{) 22} \\ -16 \\ \hline 6 \end{array}$$

* Binary subtraction:

16 21
1-5

Rück

$$\begin{array}{r}
 0 \quad 0 \\
 - 0 \\
 \hline
 0 \quad 0
 \end{array}$$

↓ ↓

Borrow difference

$$\begin{array}{r} \textcircled{2} \quad 0 \\ & -1 \\ \hline 1 & 1 \\ \downarrow \text{Barry} & \downarrow \text{dil} \end{array}$$

$$\begin{array}{r} \textcircled{3} \quad 1 \\ - 0 \\ \hline \text{Borrow } \overset{1}{0} \quad \overset{1}{1} \\ \text{different} \end{array}$$

$$\begin{array}{r} \textcircled{5} & & | \\ & - & 1 \\ \hline & & 0 \end{array}$$

Bottom difference

① Perform the following binary subtraction

$$\textcircled{a} \quad 1101 - 1000 \quad \textcircled{b} \quad 1000 - 1101$$

Ans:

$$\textcircled{a} \quad \begin{array}{r} 1101 \\ - 1000 \\ \hline 0 \end{array} \quad (0101)_2$$

Borrow \uparrow

In decimal,

$$\begin{array}{r} 13 \\ - 8 \\ \hline 5 \end{array}$$

Borrow = 0,
Answer is +ve

$$\textcircled{b} \quad \begin{array}{r} 1000 \\ - 1101 \\ \hline 1011 \end{array}$$

Borrow
1's complement
 $1011 + 1 \rightarrow 1000$
 $- (0101)_2$

In decimal,

$$\begin{array}{r} 8 \\ - 13 \\ \hline -5 \end{array}$$

Borrow = 1,
Answer is -ve
1's complement
of result is
add Borrow
gives result

* Complement of Binary Numbers:

① Write the 1's complement and 2's complement of the following $\textcircled{a} \quad 10101 \quad \textcircled{b} \quad 11100.101 \quad \textcircled{c} \quad 11100$

Ans:

$$\textcircled{a} \quad \text{Given } 10101$$

$$\begin{array}{l} 1\text{'s complement} \rightarrow 01010 \\ \text{of } 10101 \end{array}$$

$$\begin{array}{l} 2\text{'s complement} \\ \text{of } 10101 \rightarrow 01011 \end{array}$$

$$\left. \begin{array}{l} 1\text{'s complement} \\ \text{of } 10101 \rightarrow 01010 \\ 2\text{'s complement} \\ \text{of } 10101 \end{array} \right\} \begin{array}{l} + 1 \\ \hline 01011 \end{array}$$

$$\textcircled{b} \quad \text{Given } 11100.101$$

$$\begin{array}{l} 1\text{'s complement} \\ \text{of } 11100.101 \rightarrow 00011.010 \end{array}$$

$$\begin{array}{l} 2\text{'s complement} \\ \text{of } 11100.101 \rightarrow \cancel{\text{REDO}} \cancel{\text{RED}} \\ \hspace{10em} 00011.011 \end{array}$$

$$\textcircled{c} \quad \text{Given } 11100$$

$$\begin{array}{l} \cancel{00011} \rightarrow 1\text{'s complement of} \\ 11100 \\ + 1 \\ \hline 00100 \rightarrow 2\text{'s complement of} \\ 11100 \end{array}$$

① Perform the following binary Subtraction using

- ④ 1's complement ⑤ 2's complement

(i) $1110 - 1001$ (ii) $1001 - 1110$

Ans:

④ 1's complement

(i) $1110 \rightarrow$ Minuend

$-1001 \rightarrow$ Subtrahend

$\underline{\quad}$
 $1110 \rightarrow$ Minuend

$+0110 \rightarrow$ 1's complement of
Subtrahend (1001)

$\underline{0100}$

carry \rightarrow $1 +$
 $\underline{\quad}$
 $\underline{(0101)_2}$

(ii) $1001 \rightarrow$ Minuend
 $-1110 \rightarrow$ Subtrahend

$\underline{1001} \rightarrow$ Minuend

$+0001 \rightarrow$ 1's complement of
Subtrahend (1110)

↑ No carry

Take 1's complement of 1010

$- (0101)_2$

No carry,
-ve, take 1's
complement

carry, +ve,
Add carry to
result

⑤ 2's complement

(i) $1110 \rightarrow$ Minuend

$-1001 \rightarrow$ Subtrahend

$\underline{1110} \rightarrow$ Minuend

$+0111 \rightarrow$ 2's complement

of 1001 (Subtrahend)

$\underline{\underline{(0101)}_2}$

carry
(Discard)

In decimal (i) $1\frac{1}{4} - 1\frac{1}{4} = 0$

$\frac{1}{4}$	$\frac{1}{4}$
$- \frac{1}{4}$	$- \frac{1}{4}$
5	-5

(iii) $1001 \rightarrow$ Minuend

$-1110 \rightarrow$ Subtrahend

$\underline{1001} \rightarrow$ Minuend

$+0010 \rightarrow$ 2's complement
of Subtrahend

↑ No carry

∴ Take 2's complement of

1011

$\underline{- (0101)_2}$

No carry,
+ve

③ Perform the following binary subtraction using
1's complement and 2's complement

$$\textcircled{a} \quad 1111 - 1101 \quad \textcircled{b} \quad 10111 - 10101$$

Ans:

1's complement

$$\begin{array}{r} \textcircled{a} \quad 1111 \rightarrow \text{Minuend} \\ - 1101 \rightarrow \text{Subtrahend} \\ \hline 1111 \rightarrow \text{Minuend} \\ + 0010 \rightarrow 1's \text{ complement} \\ \hline \textcircled{b} \quad 00001 \quad \text{of Subtrahend} \\ \text{Count} \rightarrow 1+ \\ \hline \underline{\underline{(0010)}_2} \end{array}$$

$$\begin{array}{r} \textcircled{b} \quad 10111 \rightarrow \text{Minuend} \\ - 10101 \rightarrow \text{Subtrahend} \\ \hline 10111 \rightarrow \text{Minuend} \\ + 01010 \rightarrow 1's \text{ complement of} \\ \hline \text{Subtrahend (10101)} \\ \text{Count} \rightarrow 1+ \\ \hline \underline{\underline{(00010)}_2} \end{array}$$

2's complement

$$\begin{array}{r} \textcircled{a} \quad 1111 \rightarrow \text{Minuend} \\ - 1101 \rightarrow \text{Subtrahend} \\ \hline 1111 \rightarrow \text{Minuend} \\ + 0011 \rightarrow 2's \text{ complement} \\ \hline \textcircled{b} \quad 00010 \quad \text{of Subtrahend} \\ \text{Count} \rightarrow \\ \text{Discord} \end{array}$$

$$\begin{array}{r} \textcircled{b} \quad 10111 \rightarrow \text{Minuend} \\ - 10101 \rightarrow \text{Subtrahend} \\ \hline 10111 \rightarrow \text{Minuend} \\ + 01011 \rightarrow 2's \text{ complement of} \\ \hline \text{Subtrahend (10101)} \\ \text{Count} \rightarrow \\ \text{Discord} \end{array}$$

* Logic gates

① Not gate (Inverter) (Complementor): $(-)^{\oplus}(')$

* The output is the reverse of the input. i.e. a low-voltage input (0) is converted to a high-voltage output (1) & vice versa.

* If A' is the input, then output, $Y = \bar{A}$

* The symbol & truth table of a NOT gate is shown in fig ①.

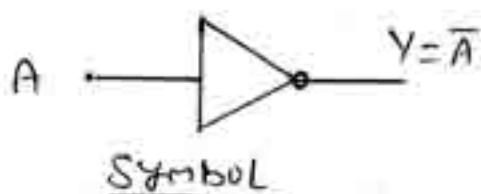


Fig ①

A	$Y = \bar{A}$
0	1
1	0

Truth table

* The input and output waveforms of a NOT gate is shown in fig ②

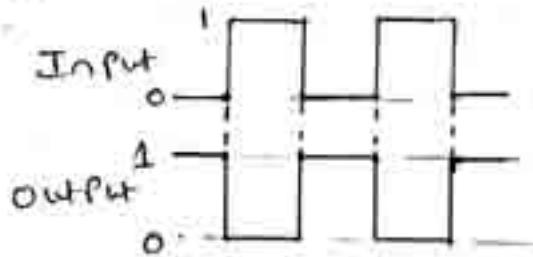


Fig ②

* The circuit diagram of a transistor based NOT gate is shown in fig ③

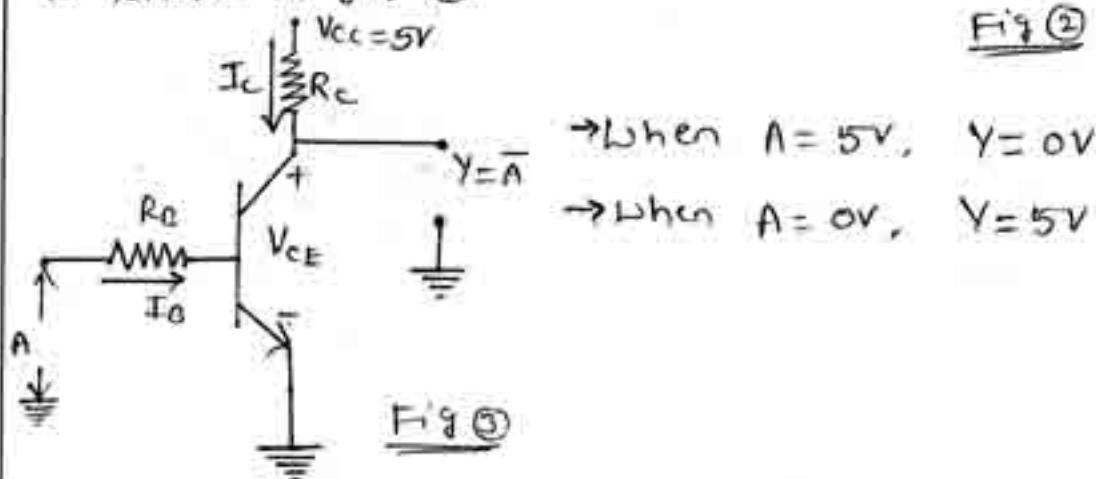


Fig ③

② AND gate: (+)

* The output is 1 only if both A and B are 1, otherwise it is zero.

* If A and B are the inputs, then the output of AND gate is,

$$Y = A \cdot B \quad \textcircled{1} \quad AB$$

* The symbol & truth table of AND gate is shown in fig ④



Symbol

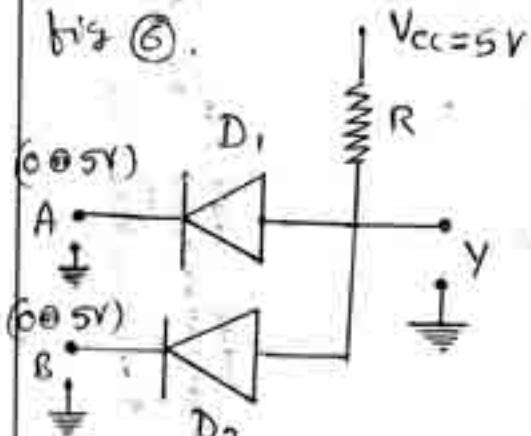
A	B	$Y = AB$
0	0	0
0	1	0
1	0	0
1	1	1

Fig(4)

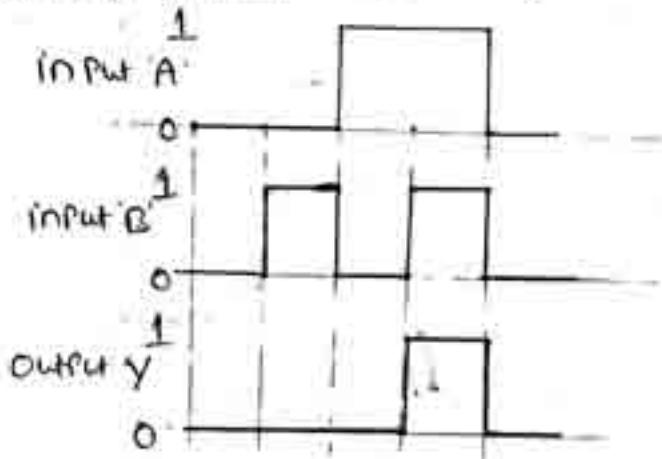
Truthtable

* The input & output waveforms of a AND gate is shown in Fig(5)

* The circuit diagram of an AND gate using diodes is shown in Fig(6).



Fig(6)



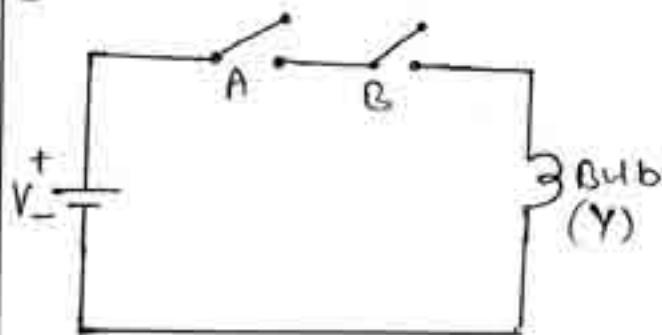
Fig(5)

* If $A=B=5V$, the diodes D_1 & D_2 do not conduct (Open circuit), then the output $Y=5V$.

* If at least one input ($A \oplus B$) is 0V, then the output $Y=0V$.

Note:

① Consider the simple electric circuit shown in Fig(7)



$A, B \rightarrow$ Switches (Closed = 1)
(Open = 0)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Fig(7)

② AND gate with N inputs (Output is 1 only when all the inputs are 1)

$$Y = A \cdot B \cdot C \cdot \dots \cdot N$$

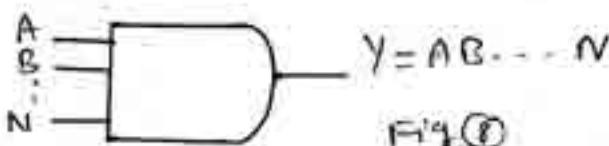


Fig ⑧

③ NAND-gate is the dual of NOR-gate & vice versa

④ OR Gate (+) :

* The output is 0 when all the inputs are zero, otherwise it is 1.

* If A and B are the inputs, then the output of OR gate is

$$Y = A + B$$

* The symbol & truth table of OR gate is shown in fig ⑨.



Fig ⑨

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

* The input and output waveforms of a OR gate is shown in fig ⑩.

* The circuit diagram of an OR gate using diodes is shown in fig ⑪

* When $A=B=0V$, the diodes D₁ & D₂ does not conduct, hence the output $Y=0V$.

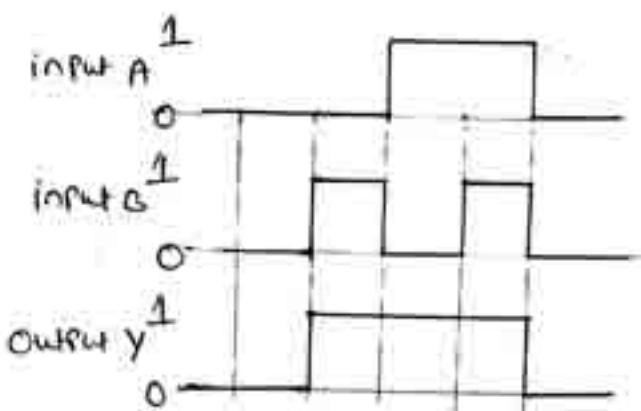
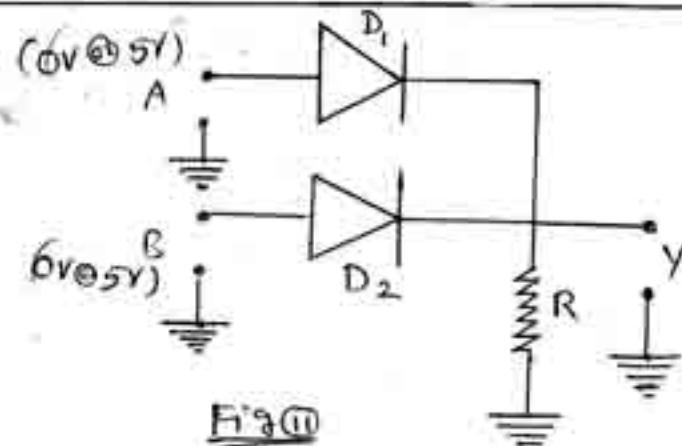


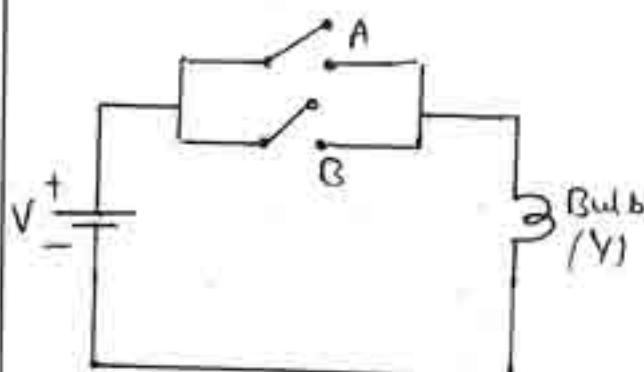
Fig ⑩

- * When either A or B both are at 5V(1), the diodes D₁ & D₂ are on (conducts) hence the output Y=1V



Note:

- ① Consider the simple electric circuit shown in fig 12

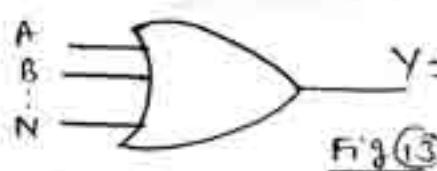


A, B → Switches (Closed = 1
Open = 0)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

- ② OR gate with N inputs (output is 0 when all the inputs are '0')

$$Y = A + B + C + \dots + N$$



③ $\bar{0} = 1$
④ $\bar{1} = 0$

(h) NAND gate:

- * The output is '0' (Low) only if both A & B are 1, otherwise it is 1.

- * If A and B are the inputs, then the output of NAND gate is,

$$Y = \overline{AB}$$

* The symbol & truth table of NAND gate is shown in fig(14)

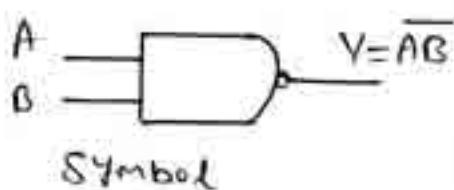


Fig (14)

A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

Truth table

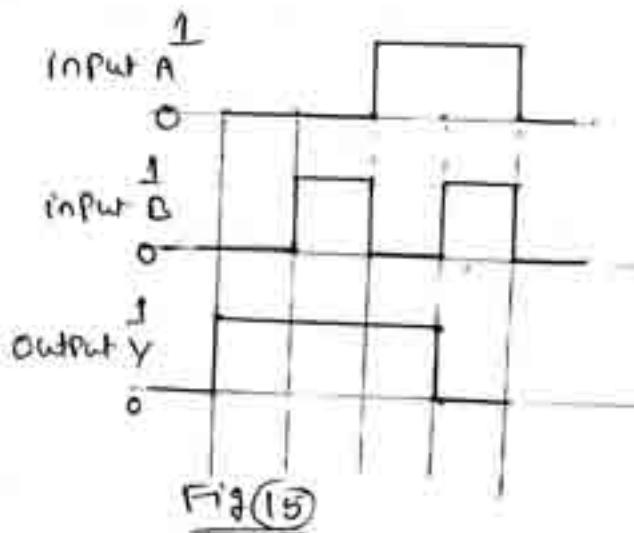
* The input & output waveforms of NAND gate is shown in fig(15)

* The output 'Y' of 'N' inputs NAND gate is,

$$Y = \overline{ABC \dots N}$$



Fig (16)



⑤ NOR gate:

* The output is '1' (high) when all the inputs are '0' (low), otherwise it is '0' (low)

* If A and B are the inputs, then the output of NOR gate is,

$$Y = \overline{A+B}$$

* The symbol & truth table of NOR gate is shown in fig(17)



Fig (17)

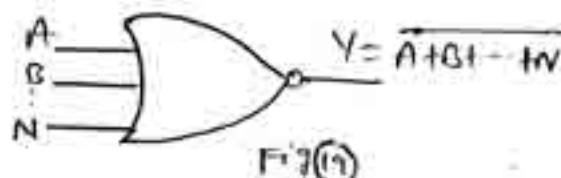
A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

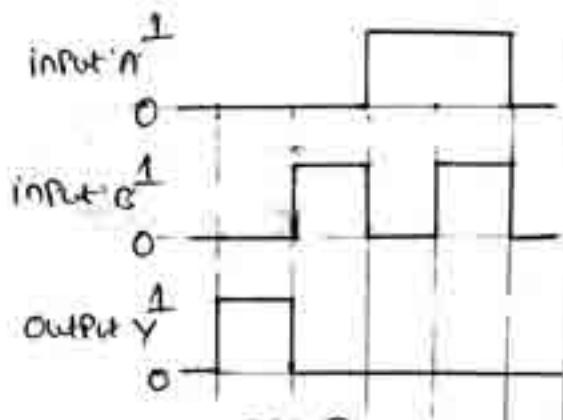
* The input and output waveforms of NOR gate is shown in Fig(18).

* The output of N inputs NOR gate is,

$$Y = \overline{A + B + \dots + N}$$



Fig(18)



⑥ EXOR gate @ Exclusive-OR gate:

* The output is '1' (high), when the inputs are different, otherwise it is '0' (low)

* If A and B are the inputs, then the output of XOR gate is,

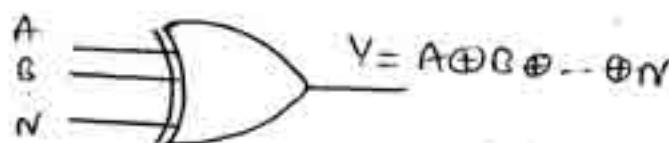
$$Y = A \oplus B = \overline{A}B + A\overline{B} (\text{or } \overline{AB} + AB)$$

* The symbol & truth table of XOR gate is shown in Fig(20)

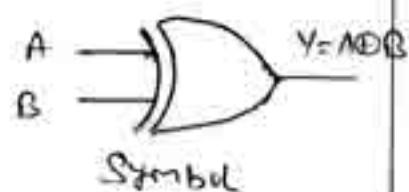
* The input & output waveforms of XOR gate is shown in Fig(21)

* The output of N inputs XOR gate is, (output is 1 when odd number of inputs are high)

$$Y = A \oplus B \oplus \dots \oplus N$$



Fig(22)

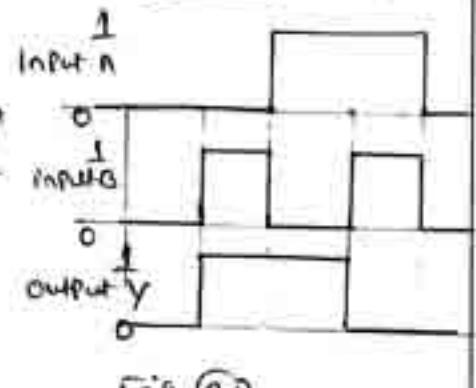


Symbol

A	B	Y = A \oplus B
0	0	0
0	1	1
1	0	1
1	1	0

Truth table

Fig(20)



Fig(21)

④ EX-NOR gate @ Exclusive-NOR gate

* The output is '1' (high) when both inputs are same [(0,0) @ (1,1)].

* If A and B are the inputs, then the output of EX-NOR gate is.

$$Y = A \oplus B = \overline{A} \overline{B} + AB (= \overline{\overline{AB} + AB})$$

* The symbol & truthtable of EX-NOR gate is shown in Fig (23)



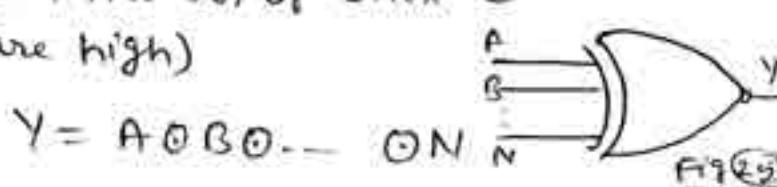
Symbol

A	B	Y = A ⊕ B
0	0	1
0	1	0
1	0	0
1	1	1

Truth table

Fig (23)

* The output 'Y' of N inputs EX-NOR gate is (output is high only when even number of ones @ all inputs are high)



* The input & output waveform of EX-NOR gate is shown in fig (24)

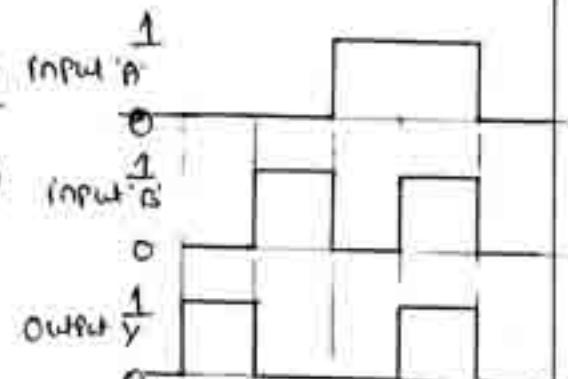


Fig (24)

* Boolean algebra theorems ⑤ (Basic Boolean Laws) :

Name	AND form	OR form
Identity Law	1 · A = A	0 + A = A
Null Law	0 · A = 0	1 + A = 1
Idempotent Law	A · A = A	A + A = A
Inverse Law	A · \bar{A} = 0	$A + \bar{A} = 1$
Commutative Law	A · B = B · A	$A + B = B + A$
Associative Law	$(AB)C = A(BC)$	$(A+B)+C = A+(B+C)$
Distributive Law	$A+BC = (A+B)(A+C)$	$A(B+C) = AB+AC$
Absorption Law	$A(A+B) = A$	$A+A\bar{B} = A$
DeMorgan's Law	$\overline{A \cdot B} = \overline{A} + \overline{B}$	$\overline{A+B} = \overline{A} \cdot \overline{B}$

Proof:

$$\textcircled{1} \quad I \cdot A = A$$

	1	2
A	I · A	
0	0	
1	1	

From $I^m \times A$ 2nd column

$$I \cdot A = A //.$$

$$\textcircled{2} \quad 0 + A = A$$

	1	2
A	0 + A	
0	0	
1	1	

From $I^m \times A$ 2nd column

$$A = 0 + A$$

$$\textcircled{3} \quad 0 \cdot A = 0$$

	1	2
A	0 · A	
0	0	
1	0	

From 2nd column
output is zero
always

$$\therefore 0 \cdot A = 0$$

$$\textcircled{4} \quad I + A = A$$

	1	2
A	I + A	
0	1	
1	1	

From 2nd column
output is
always 1

$$\therefore I + A = A //$$

$$\textcircled{5} \quad A \cdot A = A$$

	1	2
A	A · A	
0	0	
1	1	

From $I^m \times A$ 2nd column

$$A \cdot A = A //$$

$$\textcircled{6} \quad A + A = A$$

	1	2
A	A + A	
0	0	
1	1	

From $I^m \times A$ 2nd column

$$A + A = A //$$

$$\textcircled{7} \quad A \cdot \bar{A} = 0$$

	1	2	3
A	\bar{A}	$A\bar{A}$	
0	1	0	
1	0	0	

From 3rd column
output is always
0

$$\therefore A\bar{A} = 0 //$$

$$\textcircled{8} \quad A + \bar{A} = 1$$

	1	2	3
A	\bar{A}	$A + \bar{A}$	
0	1	1	
1	0	1	

From 3rd column
the output is always
1

$$\therefore A + \bar{A} = 1 //$$

$$\textcircled{9} \quad AB = BA$$

	1	2	3	4
A	B	AB	BA	
0	0	0	0	
0	1	0	0	
1	0	0	0	
1	1	1	1	

From 3rd & 4th column

$$AB = BA //$$

$$\textcircled{10} \quad A + B = B + A$$

	1	2	3	4
A	B	A+B	B+A	
0	0	0	0	
0	1	1	1	
1	0	1	1	
1	1	1	1	

From 3rd & 4th column

$$A + B = B + A //$$

$$\textcircled{11} \quad (AB)C = A(BC)$$

	1	2	3	4	5	6	7
A	B	C	AB	BC	(AB)C	A(BC)	
0	0	0	0	0	0	0	
0	0	1	0	0	0	0	
0	1	0	0	0	0	0	
0	1	1	0	1	0	0	
1	0	0	0	0	0	0	
1	0	1	0	0	0	0	
1	1	0	1	0	0	0	
1	1	1	1	1	1	1	

From 6th & 7th column

$$(AB)C = A(BC) //$$

$$(12) (A+B)+C = A+(B+C)$$

A	B	C	A+B	B+C	(A+B)+C	A+(B+C)
1	2	3	4	5	6	7
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

$$(13) A+BC = (A+B)(A+C)$$

Method 1

$$RHS = (A+B)(A+C)$$

$$= AA + AC + AB + BC$$

$$= A + AC + AB + BC$$

$$= A(1+C+B) + BC$$

$$= A+BC = \underline{LHS}$$

$$(\because AA = 1, 1+C+B = 1)$$

Method 2

A	B	C	BC	A+BC	(A+B)	(A+C)	XY
1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

From 5th & 8th Column

$$A+BC = (A+B)(A+C) //.$$

$$(15) A(A+B) = A$$

Method 1

$$LHS = A(A+B)$$

$$= AA + AB$$

$$= A + AB (\because AA = A)$$

$$= A(1+B)$$

$$= A (\because 1+B=1)$$

$$= \underline{RHS}$$

From Columns 6th & 7th,

$$(A+B)+C = A+(B+C) //$$

$$(14) A(B+C) = AB+AC$$

Method 1

$$RHS = AB+AC$$

$$= A(B+C) = LHS$$

Method 2

A	B	C	B+C (X)	AX	AB (Y)	AC (Z)	Y+Z
1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

From 5th & 8th Column,

$$A(B+C) = AB+AC //.$$

(15) Method 2

1	2	3	4
A	B	$A+B$	$A(A+B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

From 1st & 4th column,

$$A(A+B) = A \quad //.$$

(16) $A+A\bar{B}=A$ Method 1

$$\begin{aligned} LHS &= A+A\bar{B} \\ &= A(1+\bar{B}) \\ &= A \quad (\because 1+\bar{B}=1) \\ &= RHS \end{aligned}$$

Method 2

1	2	3	4
A	B	AB	$A+A\bar{B}$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

From 1st & 4th column,

$$A+A\bar{B}=A \quad //.$$

Note:

$$\textcircled{1} \quad EX-OR = \overline{EX-NOR}$$

$$\therefore A \oplus B = \overline{A \odot B}$$

$$\textcircled{2} \quad EX-NOR = \overline{EX-OR}$$

$$\therefore A \odot A = \overline{A \oplus B}$$

$$\textcircled{3} \quad \Rightarrow \overline{AB} + A\overline{B} = \overline{A}\overline{B} + A\overline{B}$$

$$\textcircled{4} \quad \overline{A}\overline{B} + A\overline{B} = \overline{A}\overline{B} + A\overline{B}$$

$$\textcircled{5} \quad \overline{A} = A$$

Proof:

1	2	3
A	\overline{A}	$\overline{\overline{A}}$
0	1	0
1	0	1

From 1st & 3rd column,

$$\overline{\overline{A}} = A \quad //.$$

* De-Morgan's theorem @ Logic

$$\textcircled{1} \quad \overline{A \cdot B} = \overline{A} + \overline{B}$$

- $\textcircled{2}$ The complement of a product is equal to the sum of the complements.

Proof:

1	2	3	4	5	6	7
A	B	\overline{A}	\overline{B}	$\overline{A} + \overline{B}$	AB	\overline{AB}
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

From 5th & 7th column, $\overline{A \cdot B} = \overline{A} + \overline{B} \quad //.$

$$② \overline{A+B} = \overline{A} \cdot \overline{B}$$

The complement of a sum is equal to the product of the complements.

Proof:

	1	2	3	4	5	6	7
A	B	\overline{A}	\overline{B}	$\overline{A} \cdot \overline{B}$	$A+B$	$\overline{A+B}$	
0	0	1	1	1	0	1	
0	1	1	0	0	1	0	
1	0	0	1	0	1	0	
1	1	0	0	0	1	0	

From 5th & 7th columns $\overline{A+B} = \overline{\overline{A} \cdot \overline{B}}$

Note:

① De Morgan's theorem for 3 Variables

$$④ \overline{ABC} = \overline{A} + \overline{B} + \overline{C} \quad ⑤ \overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

② De Morgan's theorem for 4 Variables

$$⑥ \overline{ABCD} = \overline{A} + \overline{B} + \overline{C} + \overline{D} \quad ⑦ \overline{A+B+C+D} = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$$

③ De Morgan's theorem for N Variables

$$⑧ \overline{ABCD\dots N} = \overline{A} + \overline{B} + \overline{C} + \dots + \overline{N}$$

$$⑨ \overline{A+B+C+\dots+N} = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \dots \cdot \overline{N}$$

Proof: ①

$$2^N = 2^3 = 8 \text{ combinations}$$

$$② 2^4 = 16 \text{ combinations}$$

* 2 Variables = N
 $2^N = 2^2 = 4 \text{ combinations}$

③ AND, OR, NOT gates \rightarrow Basic Gates
 ④ NAND, NOR \rightarrow Universal Gates
 (Any logic gate can be realized)

Proof of ① (a)

1	2	3	4	5	6	7	8	9
A	B	C	$A \cdot B \cdot C$	$\bar{A} \cdot \bar{B} \cdot \bar{C}$	\bar{A}	\bar{B}	\bar{C}	$\bar{A} + \bar{B} + \bar{C}$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	0	1	1
0	1	0	0	1	0	1	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

From 5th & 9th column. $\boxed{\bar{A} \cdot \bar{B} \cdot \bar{C} = \bar{A} + \bar{B} + \bar{C}}$ //

(b)

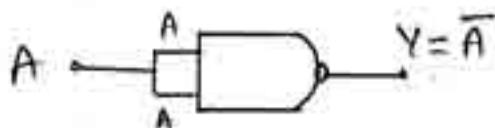
1	2	3	4	5	6	7	8	9
A	B	C	\bar{A}	\bar{B}	\bar{C}	$\bar{A} \cdot \bar{B} \cdot \bar{C}$	$\bar{A} + \bar{B} + \bar{C}$	$\bar{\bar{A} + \bar{B} + \bar{C}}$
0	0	0	1	1	1	1	0	1
0	0	1	1	1	0	0	1	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	1	0
1	0	0	0	1	0	0	1	0
1	0	1	0	1	1	0	1	0
1	1	0	0	1	0	0	1	0
1	1	1	0	0	0	0	1	0

From 9th & 9th Column. $\boxed{\bar{A} + \bar{B} + \bar{C} = \bar{A} \cdot \bar{B} \cdot \bar{C}}$ //

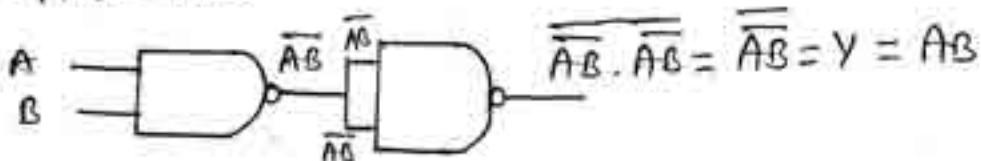
Problems:

- D) Realise the following gates using NAND gates
 ① NOT gate ② AND gate ③ OR gate ④ NOR gate
 ⑤ EX-OR gate ⑥ EX-NOR gate

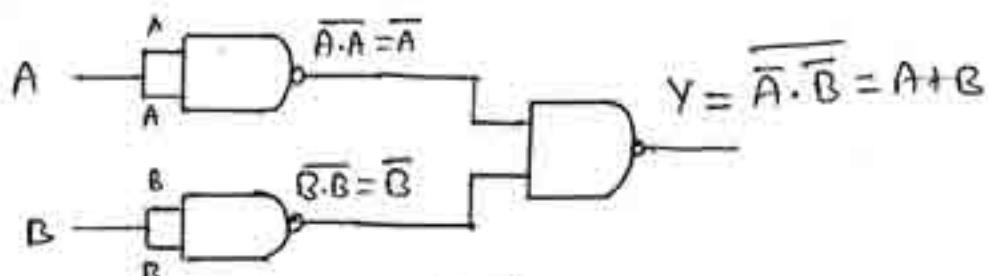
Sol: ① $Y = \bar{A} = \bar{A} \cdot A$ $(\because A \cdot A = A)$



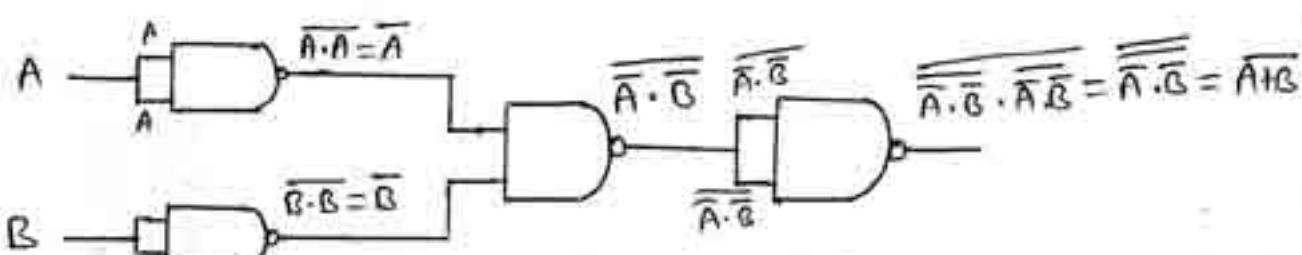
⑥ $Y = AB = \overline{\overline{AB}}$



⑦ $Y = A+B = \overline{\overline{A+B}} = \overline{\overline{A}} \cdot \overline{\overline{B}}$

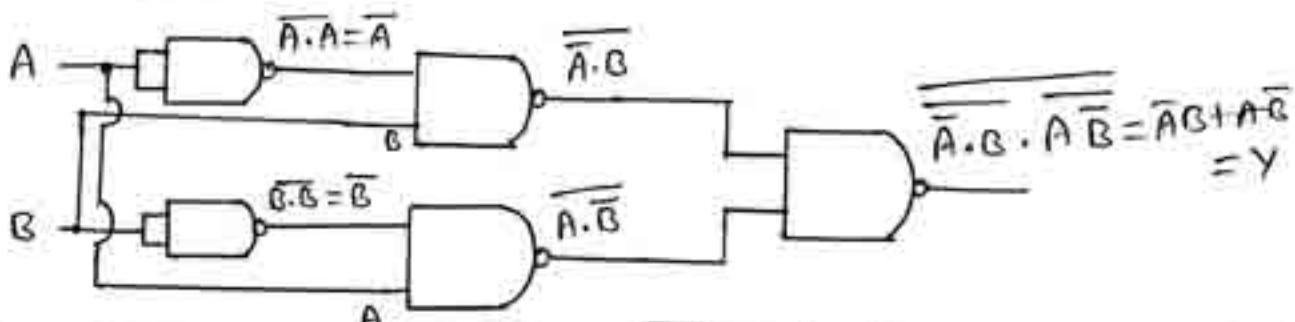


⑧ $Y = \overline{A+B} = \overline{A} \cdot \overline{B} = \overline{\overline{A} \cdot \overline{B}}$



⑨ $Y = \overline{AB} + A\overline{B} = \overline{\overline{AB} + A\overline{B}} = \overline{\overline{AB}} \cdot \overline{A\overline{B}}$ (Requires 5 NAND gates)

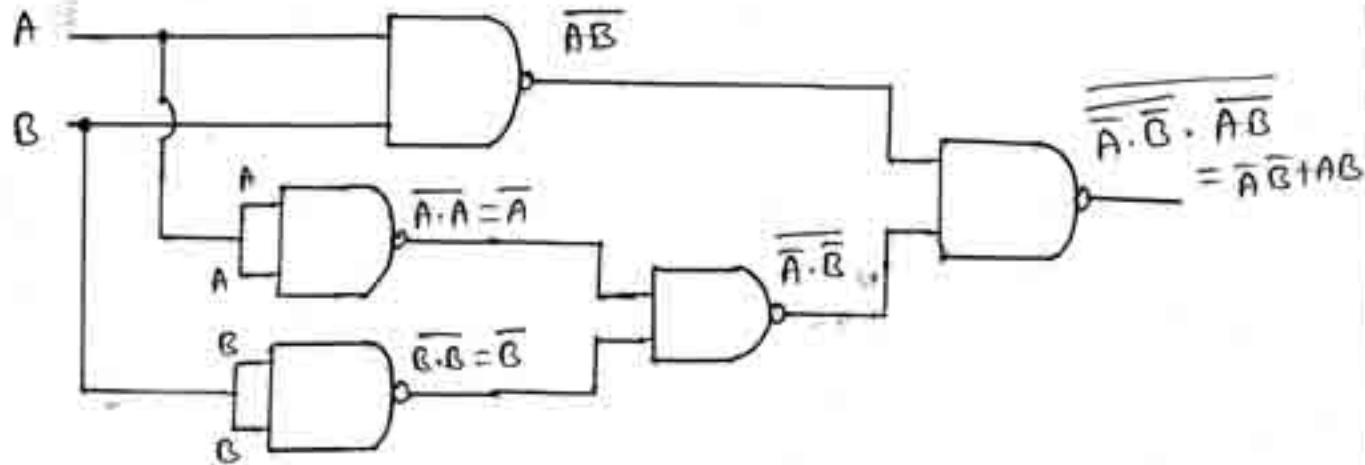
⑩ $Y = \overline{A\overline{B}} + A\overline{B} = \overline{\overline{A\overline{B}}} \cdot \overline{A\overline{B}}$ (Requires 6 NAND gates)



⑪ $Y = \overline{A}\overline{B} + A\overline{B} = \overline{\overline{A}\overline{B} + A\overline{B}} = \overline{\overline{A}\overline{B}} \cdot \overline{A\overline{B}}$ (Requires 5 NAND gates)

⑫ $Y = \overline{A}\overline{B} + A\overline{B} = \overline{\overline{A}\overline{B}} \cdot \overline{A\overline{B}} = \overline{A}\overline{B} \cdot \overline{A\overline{B}}$ (Requires 6 NAND gates)

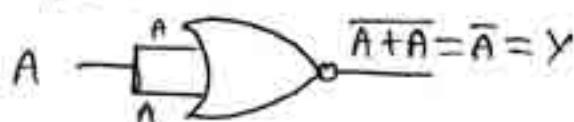
Note: EX-OR gate = EX-NOR gate & vice versa



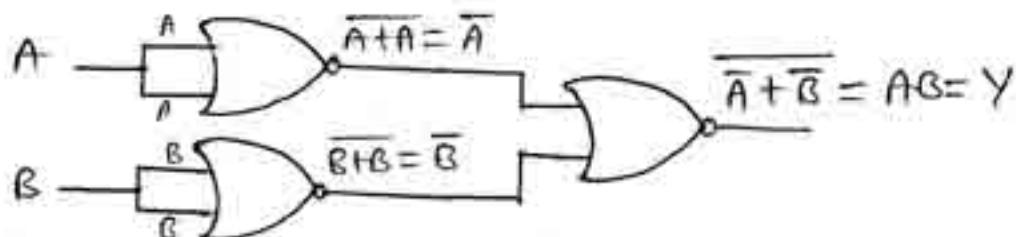
- 2) Realize the following gates using NOR Gates
 ① NOT ② AND ③ OR ④ NAND ⑤ EX-OR ⑥ EX-NOR

Ans:

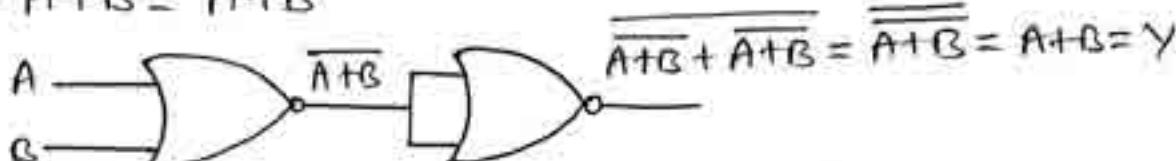
① $Y = \overline{A} = \overline{A+A} \quad (\because A+A=A)$



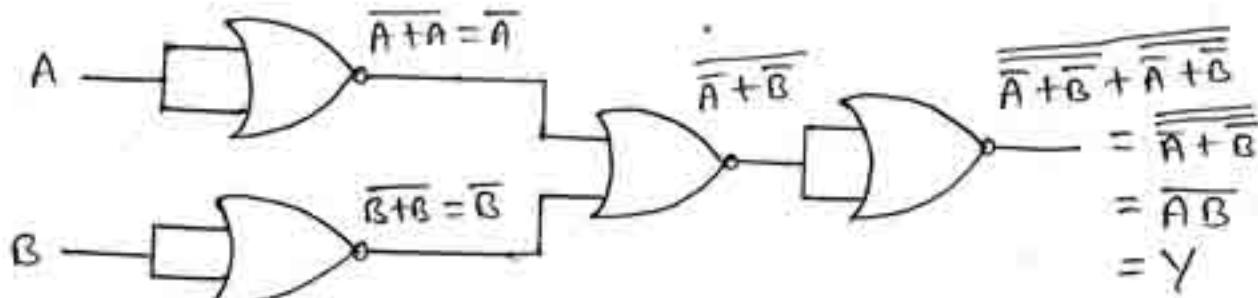
② $Y = A \cdot B = \overline{\overline{A} + \overline{B}} = \overline{\overline{A} + \overline{B}}$



③ $Y = A + B = \overline{\overline{A} + \overline{B}}$



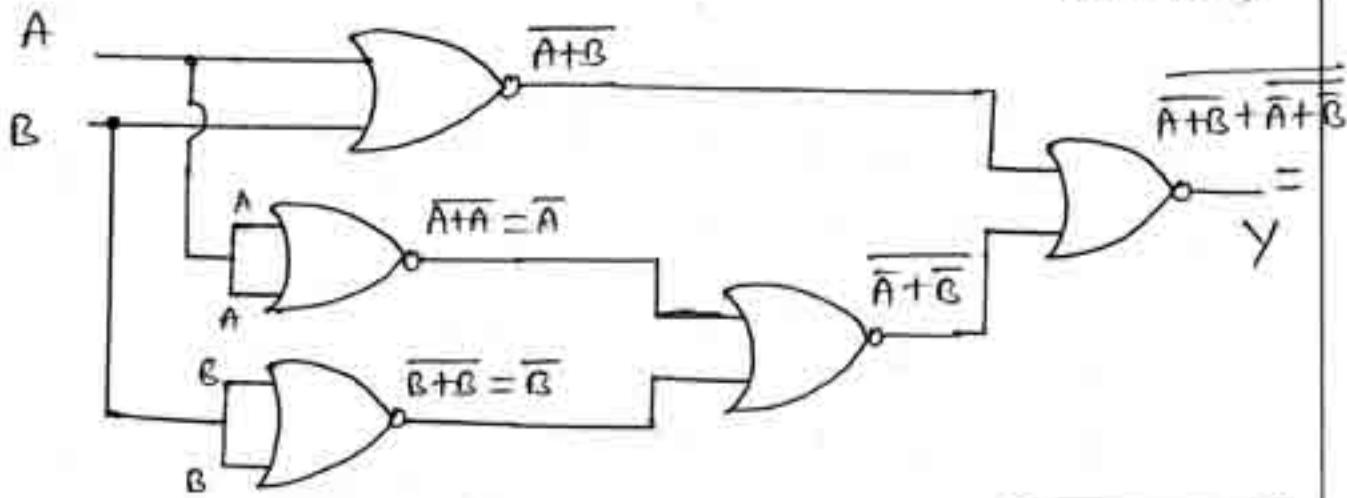
④ $Y = \overline{A} \cdot \overline{B} = \overline{A} + \overline{B} = \overline{\overline{A} + \overline{B}}$



$$\textcircled{c} \quad Y = \overline{A}B + A\overline{B} = \overline{\overline{A}\overline{B}} + \overline{A}\overline{B} = \overline{\overline{A} + \overline{B}} + \overline{A} + \overline{\overline{B}} \quad \text{Ans}$$

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$$\begin{aligned}
 \textcircled{v} \quad Y &= \overline{\overline{AB} + AB} = \overline{\overline{A}\overline{B} + \overline{A}B} = \overline{\overline{A} + \overline{B}} + \overline{\overline{A} + B} \quad (6 \text{ gates required}) \\
 &= \overline{A + B} + \overline{\overline{A} + \overline{B}} \quad (5 \text{ gates required})
 \end{aligned}$$



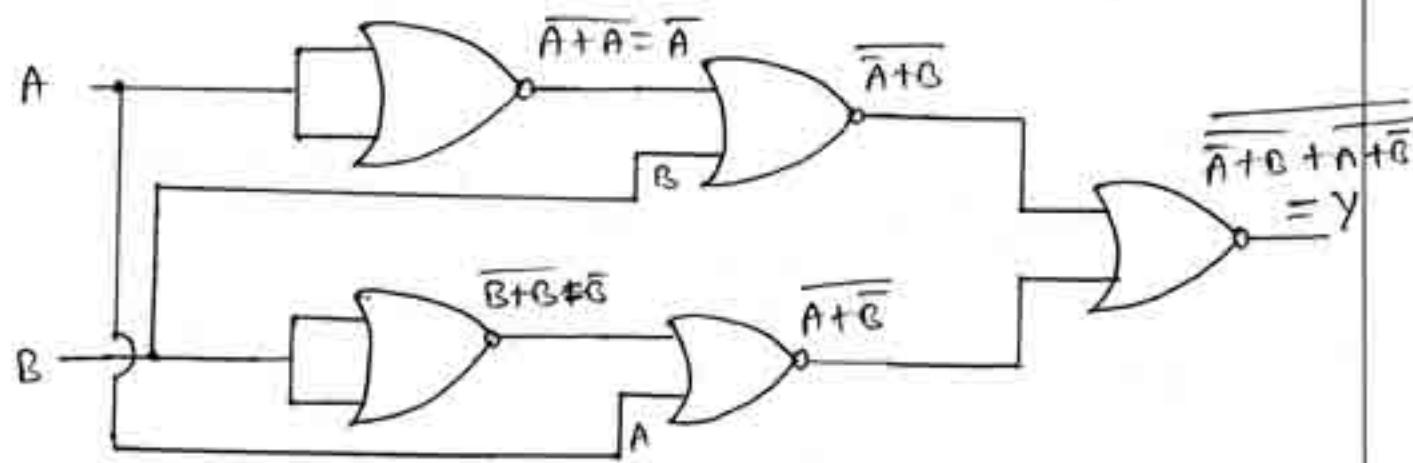
$$⑤ Y = \overline{A} \overline{B} + A\overline{B} = \overline{\overline{A} \overline{B}} + \overline{A} \overline{B} = \overline{\overline{A} + \overline{B}} + \overline{A} \overline{B} = \overline{\overline{A} + B} + \overline{A} \overline{B}$$

6

③ (6 more square)

$$Y = \overline{\overline{A}B + A\overline{B}} = \overline{\overline{A}\overline{B} + A\overline{B}} = \overline{\overline{A} + \overline{B}} + \overline{A + \overline{B}} = \overline{\overline{A} + \overline{B}} + \overline{\overline{A} + B}$$

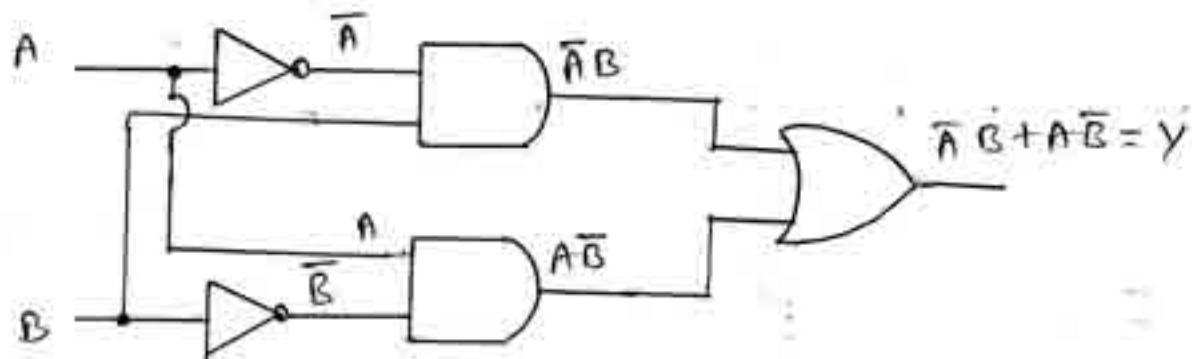
(5 gates required)



③ Realize EX-OR & EX-NOR gates using basic gates.

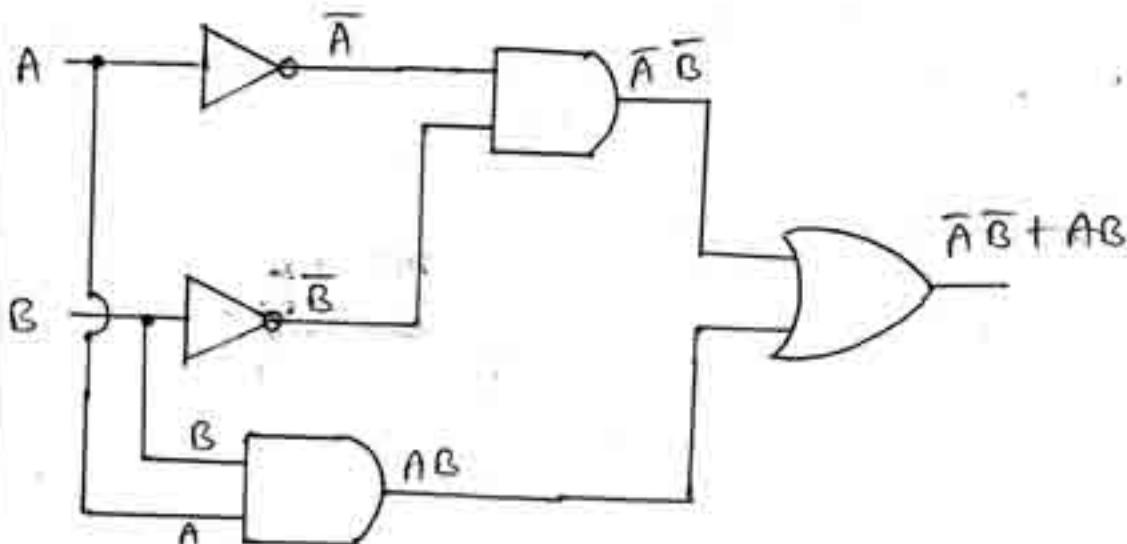
Rul: EX-OR

$$Y = \overline{A}B + A\overline{B} \quad @) \quad \overline{\overline{A}\overline{B} + A\overline{B}}$$



EX-NOR

$$Y = \bar{A}\bar{B} + AB @ \bar{A}B + A\bar{B}$$



- D) Construct the truth table for the following Boolean expressions.

$$@ Y = \overline{AB + \bar{A} + \bar{A}\bar{B}} \quad @ Y = \overline{\overline{AB} + AB} \quad @ Y = A(\bar{B} + \bar{C})$$

SOL: @

A	B	\bar{A}	AB	$\bar{A}\bar{B}$	$X+Z+P$	$Y = \overline{X+Z+P}$
0	0	1	0	1	1	0
0	1	1	0	1	1	0
1	0	0	0	1	1	0
1	1	0	1	0	1	0

$$@ Y = \overline{\overline{AB} + AB} = \overline{AB} + AB$$

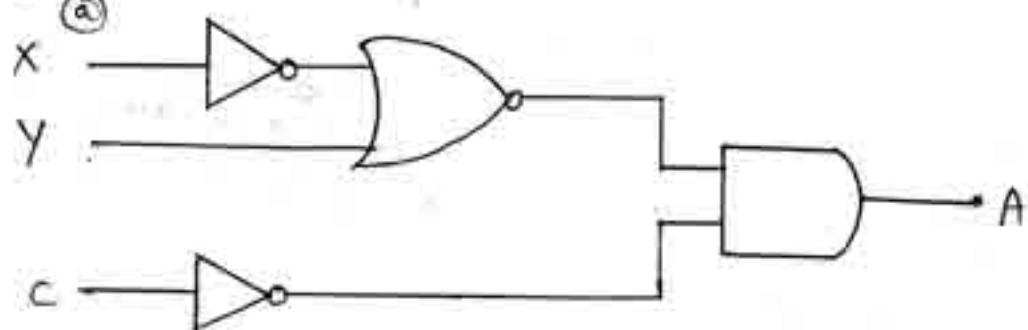
A	B	AB	$\bar{A}\bar{B}$	$Y = \overline{AB} + AB$
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1

④ $Y = A(\bar{B} + \bar{C})$

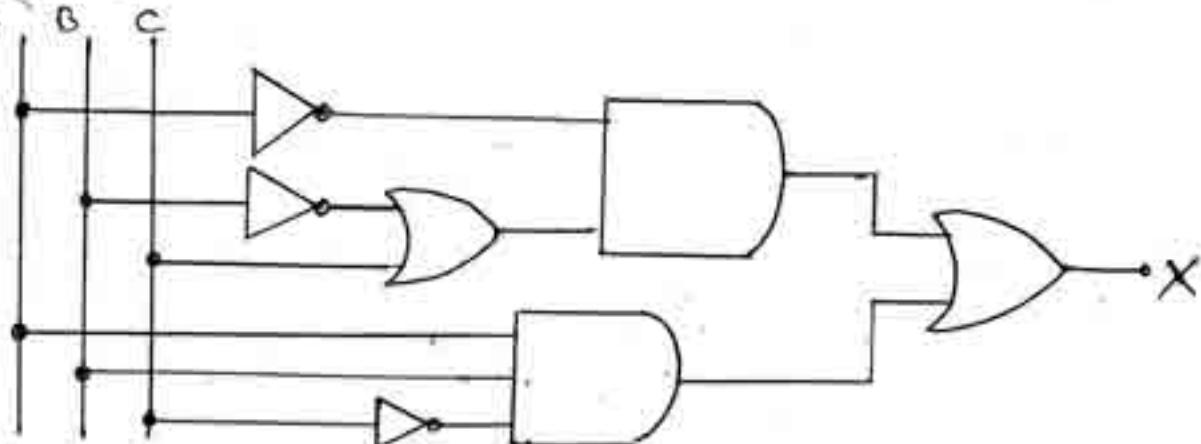
A	B	C	\bar{B}	\bar{C}	$\bar{B} + \bar{C}$	$A(\bar{B} + \bar{C})$
0	0	0	1	1	1	0
0	0	1	1	0	1	0
0	1	0	0	1	1	0
0	1	1	0	0	0	0
1	0	0	1	1	1	1
1	0	1	1	0	1	1
1	1	0	0	1	1	1
1	1	1	0	0	0	0

- ⑤ Write the Boolean expressions for the logic diagram shown below.

(a)



(b)

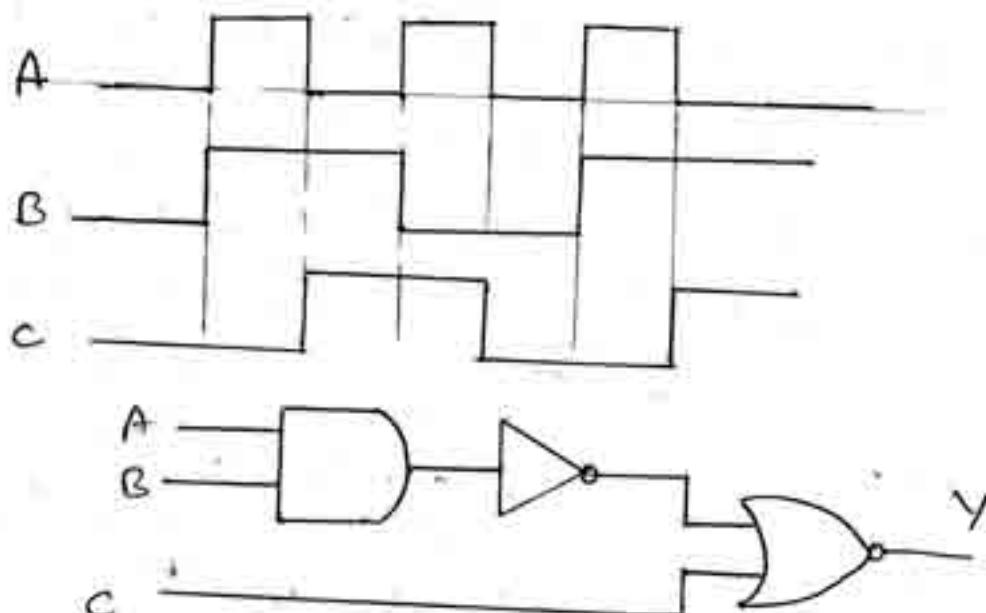


Ans:

(a) $A = (\bar{X} + Y)\bar{C}$

(b) $X = [\bar{A}(\bar{B} + C)] + A\bar{B}\bar{C}$

⑥ Draw the output waveform of the logic circuit shown for the following input waveforms.

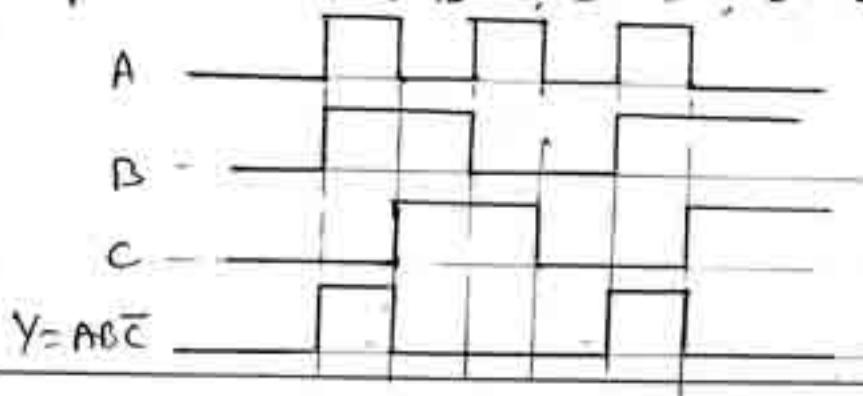


From the given logic circuit,

$$\begin{aligned} Y &= \overline{\overline{AB} + C} \\ \Rightarrow Y &= \overline{\overline{A} + \overline{B} + C} \\ &= \overline{\overline{A} \cdot \overline{B} \cdot \overline{C}} \\ &= ABC \end{aligned}$$

A	B	C	AB	\overline{C}	$Y = ABC$
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	0	1	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	1	0	0

From truth table, it is clear that output $Y = 1$ only when $A = 1$, $B = 1$, $C = 0$, otherwise output ($Y = 0$)



Q) Prove the following Boolean expression

- (a) $A + \bar{A}B = A + B$
- (b) $\bar{A} + AB = \bar{A} + B$
- (c) $A + \bar{A}B + AB\bar{C} = A + B$
- (d) $A + \bar{A}B + ABC + A\bar{C} = A + B$
- (e) $\overline{AB + \bar{A}\bar{B} + A} = 0$
- (f) $AB + \bar{A} + \overline{\bar{A}B} = 1$
- (g) $ABC + A\bar{B}C + AB\bar{C} = AB + AC$
- (h) $\bar{A}\bar{B} + \bar{A} + AB = 0$
- (i) $AB + A + A\bar{B} = A$
- (j) $\bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y} + X\bar{Y} = Y$

Sol: Method I

$$\begin{aligned}
 \text{(a) LHS} &= A + \bar{A}B \\
 &= A \cdot 1 + \bar{A}B \\
 &= A \cdot (1+B) + \bar{A}B \quad (\because 1+B=1) \\
 &= A + A\bar{B} + \bar{A}B \\
 &= A + B(A+\bar{A}) \quad (\because A+\bar{A}=1) \\
 &= A + B \cdot 1 \\
 &= A + B = \underline{\underline{\text{RHS}}}
 \end{aligned}$$

Method II (Method of Perfect induction)

	1	2	3	4	5	6
(a)	A	B	\bar{A}	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	1	0	0	0
0	1	0	1	1	1	1
1	0	0	0	0	1	1
1	1	0	0	1	1	1

From 5th & 6th column,

$$\boxed{A + \bar{A}B = A + B} \quad //$$

$$\begin{aligned}
 \text{(b) LHS} &= \bar{A} + AB = \bar{A}(1+B) + AB = \bar{A} + \bar{A}B + AB = \bar{A} + B(A+\bar{A}) \\
 &= \bar{A} + B \\
 &= \underline{\underline{\text{RHS}}}
 \end{aligned}$$

$$\text{(c) LHS} = \underline{A + \bar{A}B + A\bar{B}\bar{C}}$$

$$\begin{aligned}
 &= A(1 + B\bar{C}) + \bar{A}B \\
 &= A + \bar{A}B \quad [\because 1 + B\bar{C} = 1] \\
 &= A(1 + B) + \bar{A}B \quad [1 + B = 1] \\
 &= A + AB + \bar{A}B \quad [A + \bar{A} = 1] \\
 &= A + B(A + \bar{A}) = A + B = \underline{\underline{\text{RHS}}}
 \end{aligned}$$

- ④ LHS = $A + \bar{A}B + A\bar{B}C + A\bar{C}$
- $$= A(1 + BC + \bar{C}) + \bar{A}B \quad (\because 1 + \text{anything} = 1)$$
- $$= A + \bar{A}B$$
- $$= A + B \quad (\because A + \bar{A}B = A + B)$$
- $$= \underline{\underline{RHS}}$$
- ⑤ LHS = $\overline{AB + \bar{A}\bar{B} + A}$
- $$= \overline{AB}, \overline{\bar{A}\bar{B}}, \overline{A} \quad [\text{DeMorgan's theorem } \overline{A+B+C} = \overline{A} + \overline{B} + \overline{C}]$$
- $$= \overline{AB}, AB, \overline{A}$$
- $$= 0 \quad [\because A\bar{A} = 0 \text{ and } AB \cdot \overline{AB} = 0]$$
- $$= \underline{\underline{RHS}}$$
- ⑥ LHS = $AB + A + \bar{A}B$
- $$= AB + \overline{AB} + \bar{A}$$
- $$= 1 + \bar{A} \quad [\because A + \bar{A} = 1 \text{ and } AB + \overline{AB} = 1]$$
- $$= 1 \quad [1 + \text{anything} = 1]$$
- $$= \underline{\underline{RHS}}$$
- ⑦ LHS = $ABC + A\bar{B}C + AB\bar{C}$
- $$= AC(B + \bar{B}) + ABC$$
- $$= AC + ABC \quad (\because B + \bar{B} = 1)$$
- $$= A(C + \bar{C}B)$$
- $$= A(C + B) \quad (\because C + \bar{C}B = C + B)$$
- $$= AB + AC$$
- $$= \underline{\underline{RHS}}$$

⑤ LHS = $\overline{AB} + \overline{A} + AB = \overline{AB} \cdot \overline{A} \cdot \overline{AB} = AB \cdot A \cdot \overline{AB} = 0 = \text{RHS}$
 $(\because AB \cdot \overline{AB} = 0)$

⑥ LHS = $AB + A + AB = A(B + 1 + B) = A = \text{RHS}$
 $(\because 1 + \text{anything} = 1)$

⑦ LHS = $\overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y} + X\overline{Y}$
 $= \overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y} + X\overline{Y} \quad (\because A+A=A \text{ and } \overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y}\overline{Z} = 0)$
 $= \overline{X}\overline{Y}(\overline{Z}+1) + X\overline{Y}$
 $= \overline{X}\overline{Y} + X\overline{Y} \quad (\because \overline{Z}+1=1)$
 $= \overline{Y}(\overline{X}+X) \quad (\because X+\overline{X}=1)$
 $= \overline{Y}$
 $= \underline{\text{RHS}}$

- 8) Simplify (factorize) the following Boolean expression
- ④ $\overline{AC} + \overline{AC}$ ⑤ $AB + \overline{A}C + \overline{B}C$ ⑥ $(A + \overline{B}C)(A\overline{B} + C)$
 ⑦ $(\overline{A} + \overline{B})(\overline{A} + \overline{C})(\overline{B} + C)$ ⑧ $ABC + ABC + \overline{A}BC + A\overline{B}C$

④ Let $Y = \overline{AC} + \overline{AC} = \overline{A}C + \overline{A} + \overline{C} = \overline{A}(C \neq 1) + \overline{C}$
 $= \overline{A} + \overline{C} \quad \underline{\text{RHS}}$

⑤ Let $Y = AB + \overline{A}C + \overline{B}C$
 $= AB + C(\overline{A} + \overline{B})$
 $= AB + C \overline{A} \overline{B}$
 $= AB + C \quad \left[\begin{array}{l} \because A + \overline{A}C = A + C \\ \text{and } AB + \overline{A}B C = AB + C \end{array} \right]$

$$\textcircled{c} (A + \bar{B}C)(A\bar{B} + C)$$

$$= A\bar{A}\bar{B} + AC + A\bar{B}\bar{B}C + \bar{B}CC$$

$$= A\bar{B} + AC + A\bar{B}C + \bar{B}C \quad \left[\begin{array}{l} AA = A \\ \bar{B}\bar{B} = \bar{B} \\ CC = C \end{array} \right]$$

$$= A\bar{B} + \underline{\bar{B}C + CA} \quad (\because A+I=1)$$

$$\textcircled{d} (\bar{A} + \bar{B})(\bar{A} + \bar{C})(\bar{B} + C)$$

$$= (\bar{A} \cdot \bar{B})(\bar{A}\bar{B} + \bar{A}C + \bar{B}\bar{C} + \cancel{C})$$

$$= \bar{A}\bar{B}\bar{A}\bar{B} + \bar{A}\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{B}\bar{C}$$

$$= \bar{A}\bar{B} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

$$= \bar{A}\bar{B}(1 + C\cancel{+}\bar{C})$$

$$= \underline{\bar{A}\bar{B}}$$

$$\textcircled{e} AB + ABC + \bar{A}BC + \bar{A}\bar{B}C$$

$$= AB(\cancel{1+C}) + \bar{A}B + A\bar{B}C$$

$$= AB + \bar{A}B + A\bar{B}C$$

$$= B(A + \cancel{A}) + A\bar{B}C$$

$$= B + A\bar{B}C$$

$$= \underline{B + AC}$$

\textcircled{f} Simplify the following Boolean equation & draw the logic diagram

$$\textcircled{g} Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} \quad \textcircled{h} \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + \bar{Y}\bar{Z} + X\bar{Z}$$

$$\textcircled{i} ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

R.1

$$\textcircled{g} \text{ Given } Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$$

$$= \cancel{\bar{A}\bar{B}\bar{C}} + (\bar{A}B + A\bar{B})\bar{C}$$

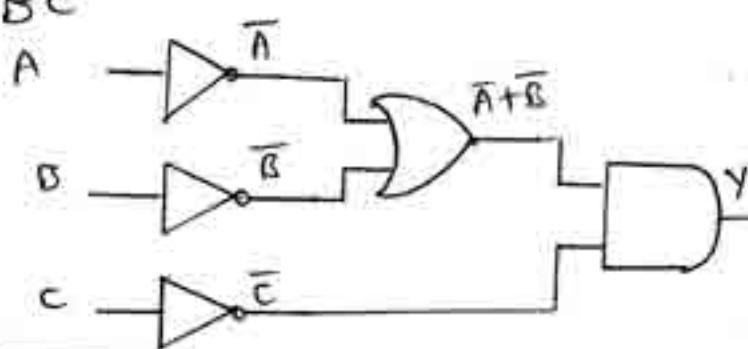
$$= \bar{A}\bar{C}(\bar{B} + B) + A\bar{B}\bar{C}$$

$$= \bar{A}\bar{C} + A\bar{B}\bar{C}$$

$$= \bar{C}(\bar{A} + A\bar{B})$$

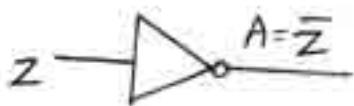
$$= \bar{C}(\bar{A} + \bar{B})$$

$$= \bar{C}\bar{A} + \cancel{\bar{C}\bar{B}}$$



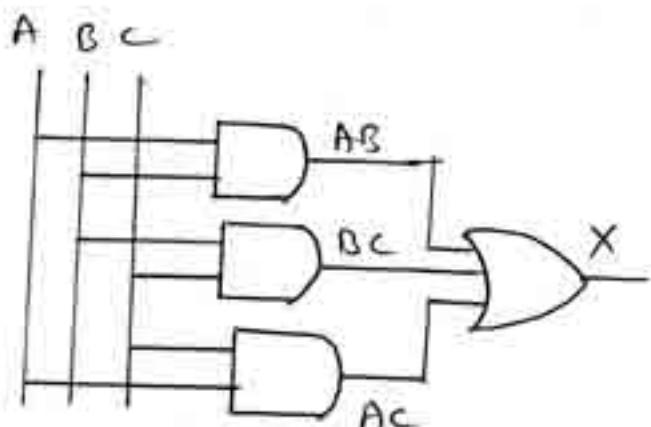
⑥ Let $A = \overline{X}\overline{Y}\overline{Z} + \underline{\overline{X}Y\overline{Z}} + \underline{\overline{Y}\overline{Z}} + X\overline{Z}$

$$\begin{aligned}&= \overline{Y}\overline{Z}(\overline{X}+1) + (\overline{X}+\overline{Y})\overline{Z} + X\overline{Z} \\&= \overline{Y}\overline{Z} + \overline{X}\overline{Z} + \overline{Y}\overline{Z} + X\overline{Z} \\&= \overline{Y}\overline{Z} + \overline{X}\overline{Z} + X\overline{Z} \\&= \overline{Y}\overline{Z} + \overline{Z}(\overline{X}+\overline{X}) \\&= \overline{Y}\overline{Z} + \overline{Z} \\&= \overline{Z}\end{aligned}$$



⑦ Let $X = \underline{ABC} + \underline{A}\overline{B}C + ABC\overline{C} + \overline{A}\overline{B}C$

$$\begin{aligned}&= ABC(C+1) + ABC\overline{C} + \overline{A}\overline{B}C \\&= AC + ABC\overline{C} + \overline{A}\overline{B}C \\&= AC + B(C\overline{C} + \overline{B}C) + \overline{A}\overline{B}C \\&= AC + B(C+1) + \overline{A}\overline{B}C \\&= AC + BC + \overline{A}\overline{B}C \\&= AC + BC(A+1) \\&= AC + BC(A+1+C) \\&= AC + BC\end{aligned}$$



⑧ Simplify & realize using only NAND-gate the following Boolean expression.

① $Y = (A+B+C)(A+B)$ ④ $Y = AB + ABC + ABC\overline{C}$

⑤ $(\overline{A}+\overline{B}C)(\overline{A}+\overline{B}+\overline{C})(A+\overline{B})$

⑥ ② $Y = (A+B+C)(A+B)$

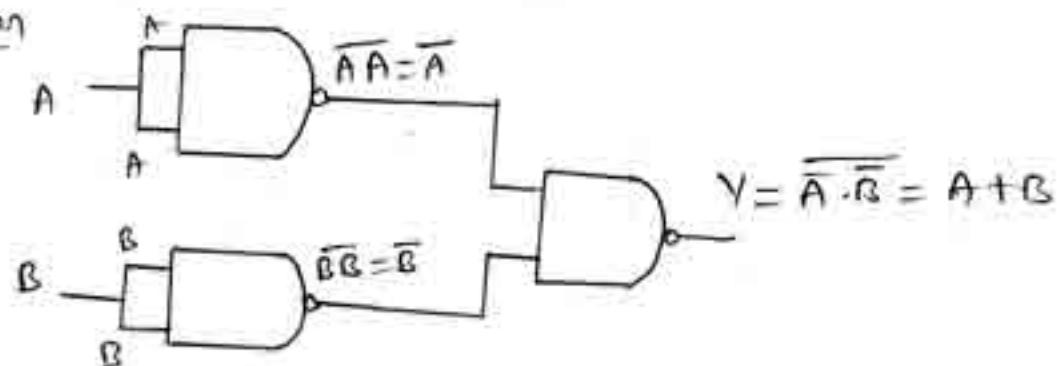
$$= AA + AB + BA + BB + CA + CB$$

$$\begin{aligned}
 &= A + A\bar{B} + B + C\bar{A} + C\bar{B} \\
 &= A(1 + \bar{B} + \bar{C}) + B(1 + \bar{C})
 \end{aligned}$$

$$Y = A + B$$

NB $Y = A + B = \overline{\overline{A+B}} = \overline{\overline{A} \cdot \overline{B}}$

Realization



(b) Given $Y = AB + ABC + ABC\bar{C}$

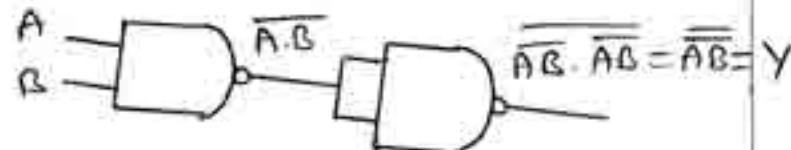
$$= AB(1 + \bar{C} + \bar{C})$$

$$Y = AB //$$

Realization

NB

$$Y = AB = \overline{\overline{AB}}$$



(c)

$$Ld X = (A + \bar{B}C)(\bar{A} + B + \bar{C})(A + \bar{C})$$

$$= (\cancel{A} + \cancel{B}^{\cancel{A}} + AB + A\bar{C} + \bar{A}\bar{B}C + B\bar{B}^{\cancel{C}} + \bar{B}\bar{C}^{\cancel{C}})(A + \bar{C})$$

$$= (AB + A\bar{C} + \bar{A}\bar{B}C)(A + \bar{C})$$

$$= AAB + A\cancel{B}^{\cancel{A}} + AAC + A\bar{B}\bar{C} + A\cancel{A}^{\cancel{C}} + \bar{A}\bar{B}\bar{C}$$

$$= AB + A\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$= AB + A\bar{C}(1 + \cancel{B}) + \bar{A}\bar{B}C$$

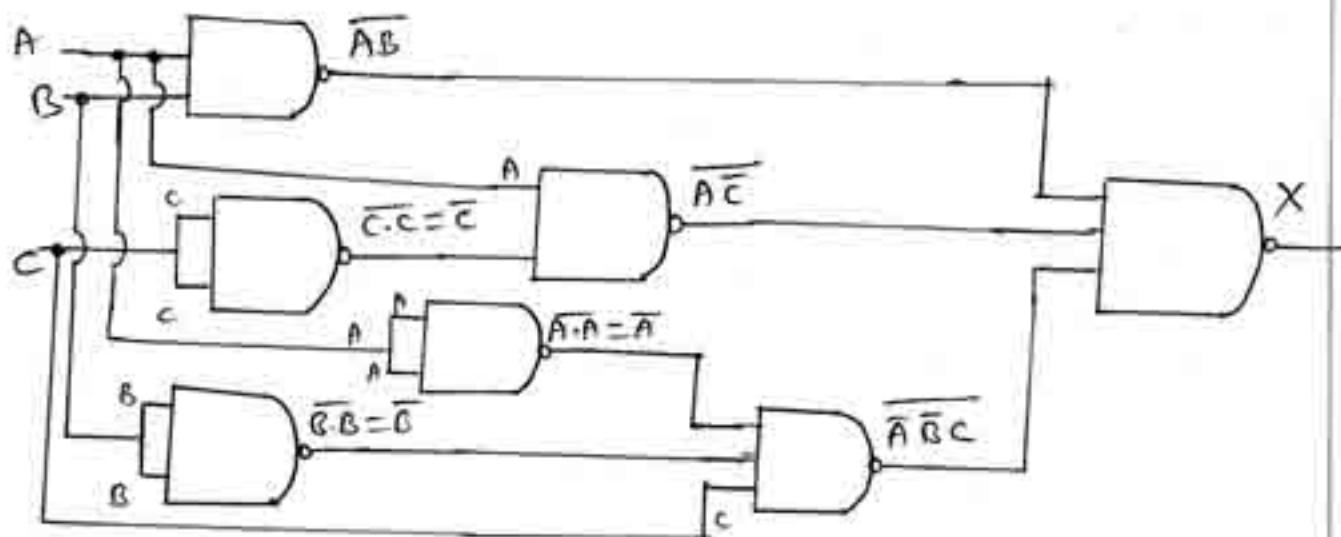
$$X = AB + A\bar{C} + \bar{A}\bar{B}C //$$

NB

$$X = AB + A\bar{C} + \bar{A}\bar{B}C = \overline{\overline{AB + A\bar{C} + \bar{A}\bar{B}C}}$$

$$X = \overline{AB} \cdot \overline{AC} \cdot \overline{\overline{A}BC}$$

Realization



- ① Simplify & implement the following expressions using only NOR gates

④ $Y = \overline{ABC}\bar{D} + \overline{ABC}D + A\overline{BC}\bar{D} + A\overline{BC}D$

⑤ $Y = \overline{(A+\bar{B}+C)(\bar{A}+B+C)(A+B)}$

⑥ $X = \overline{ABC} + A\overline{B}C + ABC$

Sol:

$$\begin{aligned} ④ Y &= \overline{ABC}\bar{D} + \overline{ABC}D + A\overline{BC}\bar{D} + A\overline{BC}D \\ &= \overline{BC}(\overline{A}\bar{D} + \overline{AD} + A\bar{D} + AD) \\ &= \overline{BC}[\overline{A}(\bar{D} + D) + A(\bar{D} + D)] \\ &= \overline{BC}(A + \bar{A}) \end{aligned}$$

$$Y = \overline{BC} //$$



NOR $Y = \overline{B}\overline{C} = \overline{B+C}$

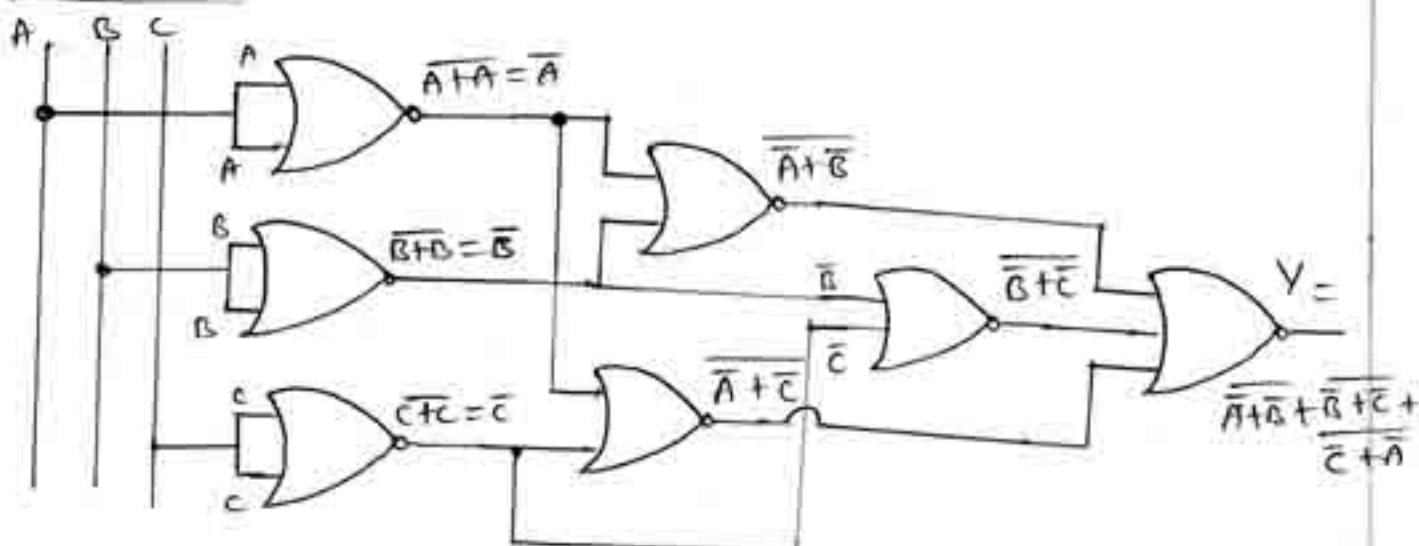
$$\begin{aligned}
 ⑤ Y &= \overline{(A+B+C)(\bar{A}+B+C)(A+\bar{C})} \\
 &= \overline{(A\bar{A}^{\text{d}} + AB + AC + \bar{A}\bar{B} + B\bar{B}^{\text{d}} + \bar{B}C + \bar{A}C + BC + CC)(A+B)} \\
 &= \overline{(AB + AC + \bar{A}\bar{B} + \bar{B}C + \bar{A}C + BC + C)(A+B)} \\
 &= \overline{(AB + \bar{A}\bar{B} + C(A + \bar{B} + \bar{A}^{\text{d}} + B + 1))(A+B)} \\
 &= \overline{(AB + \bar{A}\bar{B} + C)}(A+B) \\
 &= \overline{AAB + ABC + A\bar{A}\bar{B}^{\text{d}} + \bar{A}B\bar{B}^{\text{d}} + CA + CB} \\
 &= \overline{AB + A\bar{B} + CA + CB}
 \end{aligned}$$

$$Y = \boxed{\overline{AB + BC + CA}}$$

No^b

$$\begin{aligned}
 Y &= \overline{\overline{AB} + \overline{BC} + \overline{CA}} \\
 &= \overline{\overline{A} + \overline{B}} + \overline{\overline{B} + \overline{C}} + \overline{\overline{C} + \overline{A}}
 \end{aligned}$$

Realization



$$\textcircled{c} \quad Y = \overline{ABC} + A\overline{B}C + ABC$$

$$= \overline{ABC} + AC(\overline{B} + \overline{C})$$

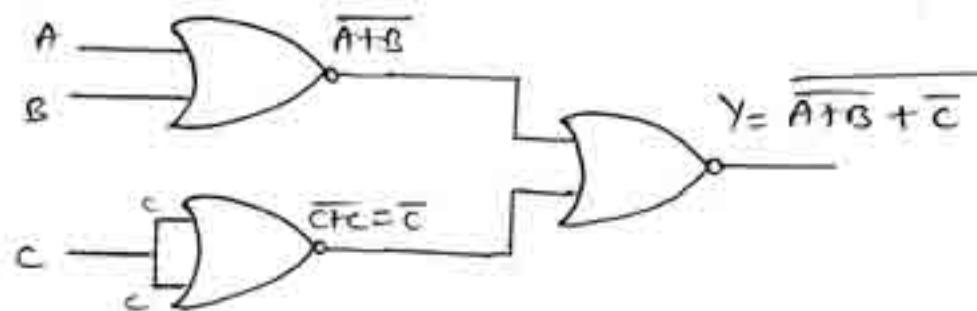
$$= \overline{ABC} + AC$$

$$= C(\overline{AB} + A) = C(A + B) \otimes CA + CB //$$

(46)

$$\begin{aligned} \text{NOR } Y &= \overline{\overline{C}(A+B)} @ Y = \overline{\overline{CA} + \overline{CB}} \\ &= \overline{\overline{C} + \overline{A+B}} \\ &= \overline{\overline{C} + \overline{A} + \overline{C} + \overline{B}} \\ &\quad (3 \text{ NOR gates required}) \\ &\quad (7 \text{ NOR gates required}) \end{aligned}$$

Realization



* Half adder:

Definition: A Logic circuit which adds two binary variables (two bits), yields a carry but does not accept carry from another circuit (adder) is called a half adder.

Block diagram



Fig①

Boolean expressions for Sum & Carry

Sum is 1 when $A=1, B=0$

& 0 when $A=0, B=1$

$$S = \overline{A}B + A\overline{B}$$

$$S = A \oplus B$$

Truth table

The truth table of a half-adder with inputs A and B and outputs sum 'S' & carry 'C' is shown in fig②

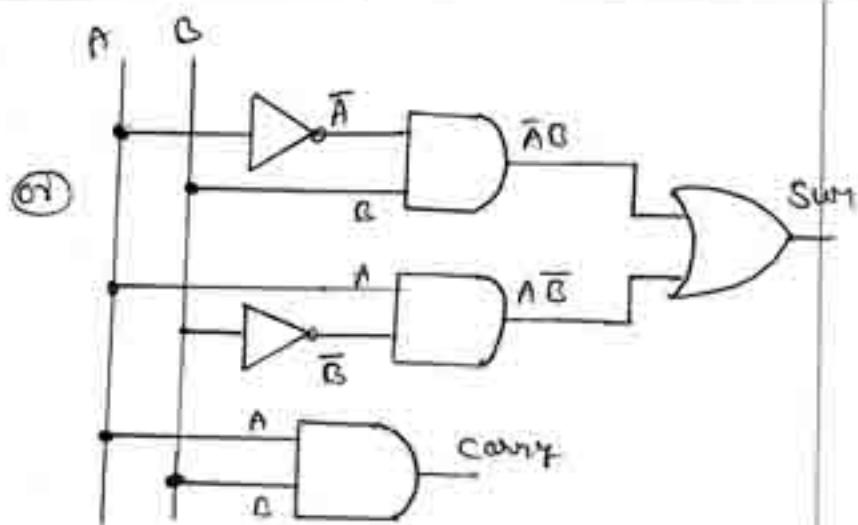
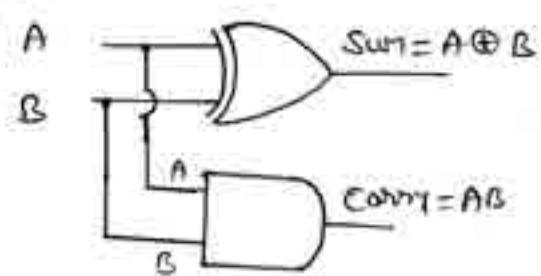
INPUTS		OUTPUTS	
A	B	SUM S	CARRY C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Fig②

Carry is 1 when $A=B=1$

$$C = AB$$

Implementation:



Limitation:

Addition of three bits cannot be done.

* Full adder:

Definition: A Logic circuit which adds two binary numbers (two bits), accept a carry and yield a carry if called Full adder.

Block diagram



Fig ①

Boolean expression for Sum(S) & Carry(Cout)

Sum is 1 when $A=0, B=0, C_{in}=1$,
 $A=0, B=1, C_{in}=0$, $A=1, B=0, C_{in}=0$
 & $A=B=C_{in}=1$

Truth table

The truth table of a full adder with inputs A, B, C_{in} and output Sum's & carry Cout is shown in fig ②

← inputs → ← outputs →

A	B	C_{in}	SUM S	Carry Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\therefore S = \overline{A} \overline{B} C_m + \overline{A} B \overline{C}_m + A \overline{B} \overline{C}_m + A B C_m \\ = (\overline{A} \overline{B} + A C_m) C_m + (\overline{A} B + A \overline{B}) \overline{C}_m \quad \text{--- (1)}$$

$$\text{Let } X = \overline{A} B + A \overline{B} \Rightarrow \overline{X} = \overline{A} \overline{B} + A B \quad \text{--- (2)} \quad [X = A \oplus B]$$

Using (2) & (3) in (1), we get

$$S = \overline{X} C_m + X \overline{C}_m$$

$$S = X \oplus C_m$$

$$\Rightarrow S = A \oplus B \oplus C_m // \quad \text{--- (1)}$$

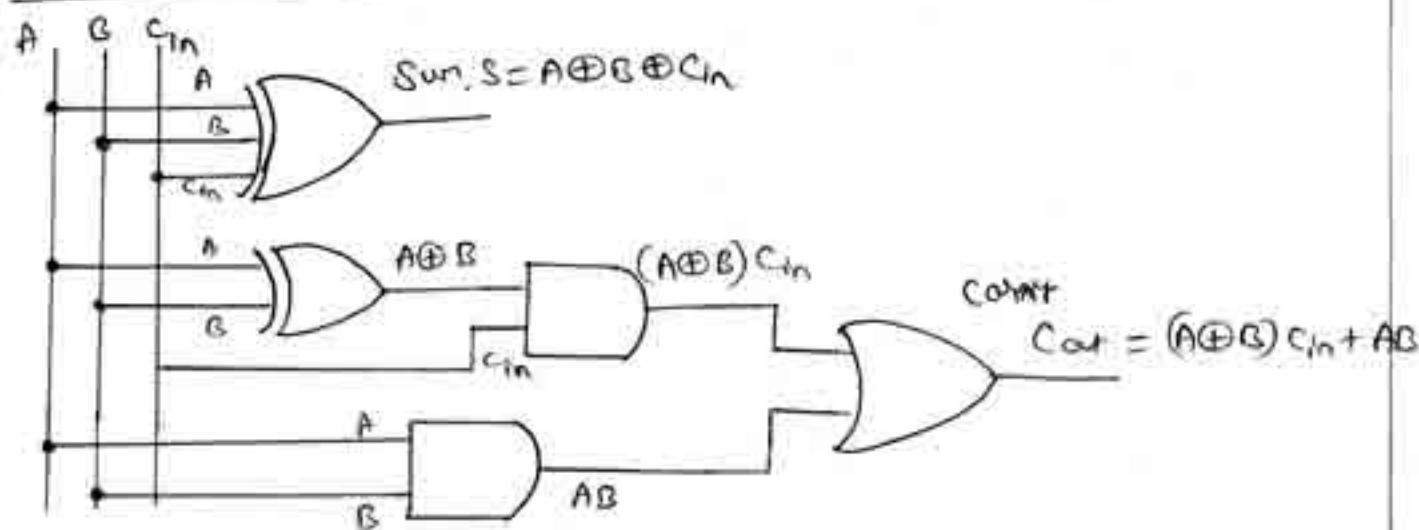
CARRY IF 1 WHEN $A=0, B=1, C_m=1, A=1, B=0, C_m=1,$
 $A=1, B=1, C_m=0 \ Leftrightarrow A=B=C_m=1$

$$\therefore C_{out} = \overline{\underset{out}{AB}} C_m + A \overline{B} C_m + A B \overline{C}_m + A B C_m$$

$$C_{out} = (A \overline{B} + A B) C_m + A B (\overline{C}_m + C_m)$$

$$C_{out} = (A \oplus B) C_m + A B \quad \text{--- (2)}$$

Implementation:



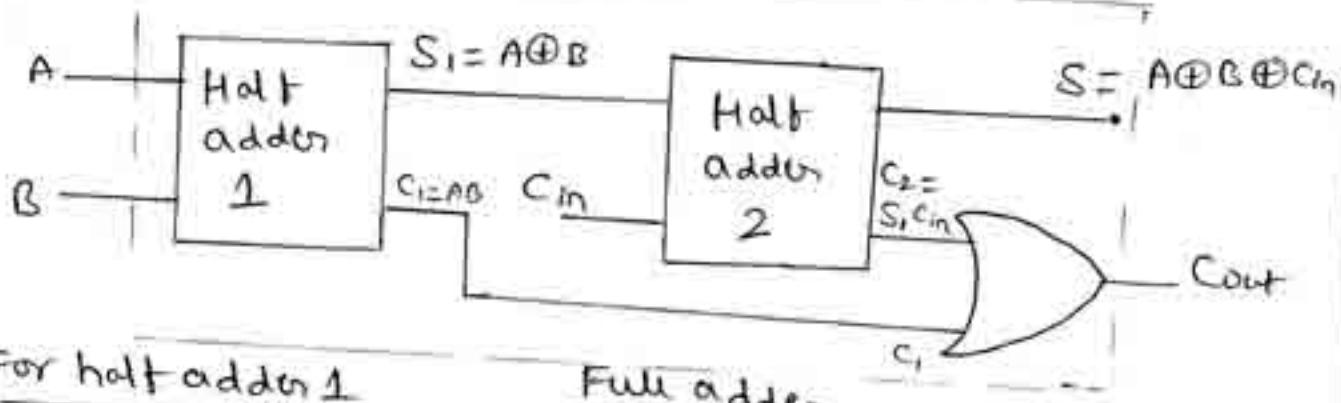
Advantage:

Addition of three bits can be done

* Implementation of Full adder using half-adder

(1) Implementation of Full adder using half-adder & NOR gate

Implementation of full adder using half adder
is shown below (Block diagram)



For half adder 1

Full adder

$$\text{Sum}, S_1 = A \oplus B \quad \text{---(3)}$$

$$\text{Carry}, C_1 = AB \quad \text{---(4)}$$

For half adder 2:

(From (3) & (4))

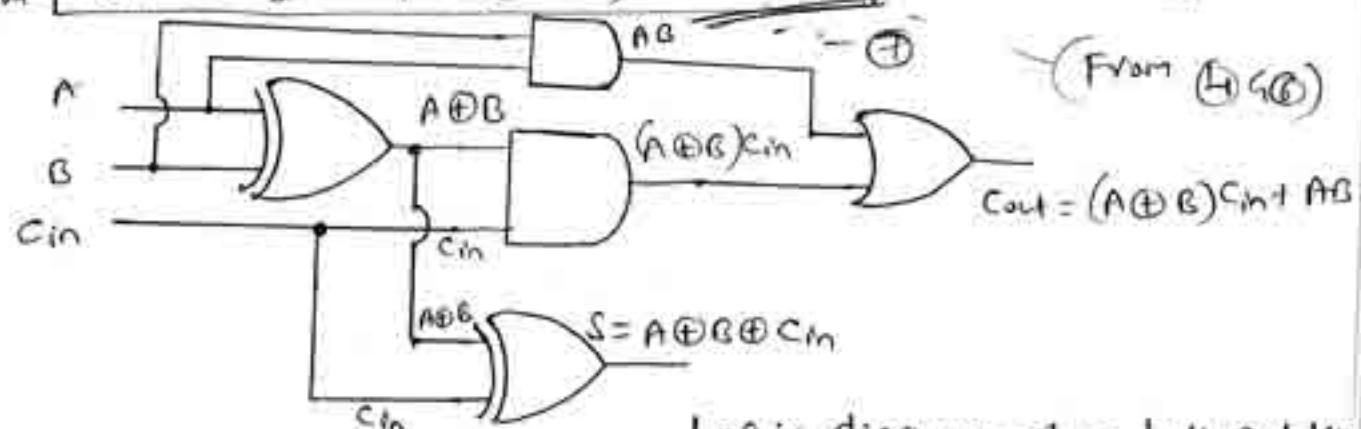
$$\text{Sum}, [S = S_1 \oplus C_{in} = A \oplus B \oplus C_{in}] \quad \text{---(5)} \quad [\text{Same as (1)}]$$

$$\text{Carry}, C_2 = S_1 C_{in} = (A \oplus B) C_{in} \quad \text{---(6)}$$

(From (3) & (5))

DR gate output

$$\text{Cout} \quad \text{Cout} = C_2 + C_1 = (A \oplus B) C_{in} + AB \quad [\text{Same as (2)}]$$



Logic diagram of a full adder

- * Properties of logic gates & Characteristics
- ① Noise Margin ② Fan-in ③ Fan-out
 - ④ propagation delay ⑤ power dissipation.

① For each element x in a Boolean algebra

$$\textcircled{a} \quad x+1=1 \quad \textcircled{b} \quad x \cdot 0=0$$

Rul: \textcircled{a}

$$\text{LHS} = x+1$$

$$= 1 \cdot (x+1)$$

$$= (x+\bar{x})(x+1)$$

$$= x + (\bar{x} \cdot 1)$$

$$= x + \bar{x}$$

$$= 1$$

$$\underline{= RHS}$$

\textcircled{b}

$$\text{LHS} = x \cdot 0$$

$$= 0 + (x \cdot 0)$$

$$= (x \cdot \bar{x}) + (x \cdot 0)$$

$$= x \cdot (\bar{x} + 0)$$

$$= x \cdot \bar{x}$$

$$= 0$$

$$\underline{= RHS}$$

② For each element x in a Boolean algebra

$$\textcircled{a} \quad x+x=x \quad \textcircled{b} \quad xx=x \quad \textcircled{c} \quad \bar{\bar{x}}=x$$

Rul:

$$\textcircled{a} \quad \text{LHS} = x+x$$

$$= (x+x) \cdot 1$$

$$= (x+x) \cdot (x+\bar{x})$$

$$= x+x\bar{x}$$

$$= x+0$$

$$= x$$

$$\underline{= RHS}$$

$$\textcircled{b} \quad \text{LHS} = xx$$

$$= x \cdot x + 0$$

$$= x \cdot x + 0 \cdot \bar{x}$$

$$= x \cdot (x+\bar{x})$$

$$= x \cdot 1$$

$$= x$$

$$\underline{= RHS}$$

$$\textcircled{c} \quad \text{LHS} = \bar{\bar{x}}$$

$$= \bar{x} + 0$$

$$= \bar{x} + x\bar{x}$$

$$= [\bar{x} + x][\bar{x} + \bar{x}]$$

$$= (x+\bar{x})(\bar{x}+\bar{x})$$

$$= (x+\bar{x}) \cdot 1$$

$$\rightarrow = (x+\bar{x})(x+\bar{x})$$

$$= x + \bar{x} \cdot \bar{x}$$

$$= x + 0$$

$$= x$$

$$\underline{= RHS}$$

- ③ For each pair of elements x and y in a Boolean algebra. ④ $x + \bar{x}y = x$ ⑥ $x(x+y) = x$

R.H.S.:

$$\begin{aligned} \textcircled{4} \quad & x + \bar{x}y = x \cdot 1 + \bar{x}y & \textcircled{6} \quad & \text{LHS} = x(x+y) \\ & = x(1+y) & & = x \cdot x + x \cdot y \\ & = x \cdot 1 & & = x + \bar{x}y \\ & = x & & = x(x+1) \\ & \underline{\underline{= RHS}} & & = x \cdot 1 \\ & & & = x \\ & & & \underline{\underline{= RHS}} \end{aligned}$$

- ④ For each pair of elements x and y in a Boolean algebra
 ④ $\bar{x} + \bar{x}y = x+y$ ⑥ $x(\bar{x}+y) = xy$

R.H.S.: ④

$$\begin{aligned} \text{LHS} &= \bar{x} + \bar{x}y \\ &= (\bar{x} + \bar{x})(x+y) \\ &= 1 \cdot (x+y) \\ &= x+y \\ & \underline{\underline{= RHS}} \end{aligned} \quad \left| \begin{array}{l} \textcircled{6} \quad \text{LHS} = x(\bar{x}+y) \\ = x \cdot \bar{x} + x \cdot y \\ = 0 + xy \\ = xy \\ = \text{RHS} \end{array} \right.$$

- ⑤ For every $x, y, z \in L$ in a Boolean algebra

$$\textcircled{4} \quad x + (y+z) = (x+y)+z \quad \textcircled{6} \quad x(yz) = (xy)z$$

R.H.S.: ④

$$\text{Let } A = x + (y+z) \quad \& \quad B = (x+y) + z.$$

$$\text{Now } xA = xA$$

$$= x[x + (y+z)]$$

$$= x$$

$$\& \quad xB = xB$$

$$= x[(x+y)+z]$$

$$= x(x+y) + xz$$

$$= xy + xz$$

$$= x$$

$$\therefore xA = xB = x \quad \text{--- } \oplus$$

Now $\bar{x}A = \bar{x}A$

$$= \bar{x}[x + (y+z)]$$

$$= \bar{x}x + \bar{x}(y+z)$$

$$= x\bar{x} + \bar{x}(y+z)$$

$$= 0 + \bar{x}(y+z)$$

$$= \bar{x}(y+z)$$

$$\bar{x}B = \bar{x}B$$

$$= \bar{x}[(x+y) + z]$$

$$= \bar{x}(x+y) + \bar{x}z$$

$$= (\bar{x}x + \bar{x}y) + \bar{x}z$$

$$= (0 + \bar{x}y) + \bar{x}z$$

$$= \bar{x}y + \bar{x}z$$

$$= \bar{x}(y+z)$$

$$\therefore \bar{x}A = \bar{x}B = \bar{x}(y+z) \quad \text{--- } \textcircled{1}$$

Now, $xA + \bar{x}A = xA + \bar{x}A$

$$xA + \bar{x}A = xB + \bar{x}B \quad [\text{using } \textcircled{1}]$$

$$Ax + A\bar{x} = Bx + B\bar{x}$$

$$A(x + \bar{x}) = B(x + \bar{x})$$

$$A \cdot 1 = B \cdot 1$$

$$A = B$$

$$\Rightarrow [x + (y+z) = (x+y) + z] //$$

Q) For each pair of elements x and y in a Boolean Algebra

a) $\overline{x+y} = \bar{x}\bar{y}$ b) $\overline{xy} = \bar{x} + \bar{y}$

Rule: b.i.e For every x in a Boolean algebra there is a unique \bar{x} such that $x + \bar{x} = 1$ and $x \cdot \bar{x} = 0$

a) It is sufficient to s.t $\bar{x}\bar{y}$ is the complement

of xy , i.e $(x+y) + (\bar{x}\bar{y}) = 1$ & $(x+y)(\bar{x}\bar{y}) = 0$

Now $(x+y) + (\bar{x}\bar{y}) = [(x+y) + \bar{x}] [(\bar{x}\bar{y}) + \bar{y}]$

$$\begin{aligned}
 &= [(\gamma + x) + \bar{\gamma}] [(\alpha + \gamma) + \bar{x}] \\
 &= [\gamma + (\alpha + \bar{\gamma})] [x + (\gamma + \bar{x})] \\
 &= (\gamma + 1)(x + 1) \\
 &= 1 \cdot 1 \\
 &= 1
 \end{aligned}$$

Also, $(\alpha + \gamma)(\bar{x} \bar{y}) = (\bar{x} \bar{y})(\gamma + \alpha)$

$$\begin{aligned}
 &= (\bar{x} \bar{y})\alpha + (\bar{x} \bar{y})\gamma \\
 &= (\bar{y} \bar{x})\alpha + (\bar{x} \bar{y})\gamma \\
 &= \bar{y}(\alpha \bar{x}) + \bar{x}(\gamma \bar{y}) \\
 &= \bar{y} \cdot 0 + \bar{x} \cdot 0 \\
 &= 0 + 0 \\
 &= 0
 \end{aligned}$$

⑤

- Q) Apply DeMorgan's theorem to each expression.
- ④ $\overline{(A+B)} + \overline{C}$ ⑤ $(\overline{A} + B) + \overline{CD}$ ⑥ $(A+B)\overline{C}\overline{D} + E + \overline{F}$

Sol:

$$\textcircled{4} \quad \overline{(A+B)} + \overline{C} = (\overline{A} + \overline{B}) \cdot \overline{\overline{C}} = (A+B)C//$$

$$\textcircled{5} \quad \overline{(\overline{A} + B) + \overline{CD}} = (\overline{\overline{A} + B}) \cdot \overline{\overline{CD}} = (\overline{A} + B)CD//$$

$$\textcircled{6} \quad \overline{(A+B)\overline{C}\overline{D} + E + \overline{F}} = (\overline{A} + \overline{B})\overline{C}\overline{D} \cdot \overline{E} \cdot \overline{\overline{F}}$$

$$= [(\overline{A} + \overline{B}) + \overline{\overline{C}} + \overline{\overline{D}}] \overline{E} \overline{F}$$

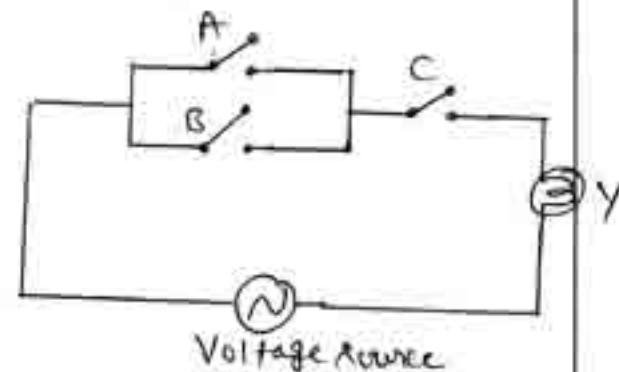
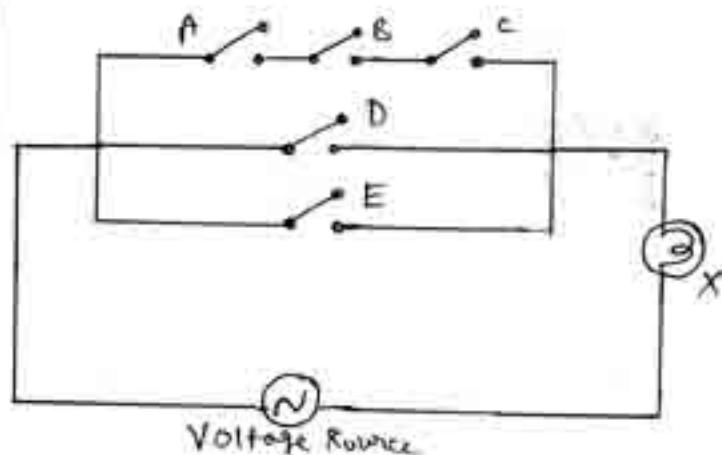
$$= (\overline{A}\overline{B} + C + D) \overline{\overline{E}} \overline{F}$$

- Q) Write a switching circuit for the following Boolean expression. ④ $X = A BC + D + E$ ⑤ $Y = (A + B) C$

Sol:

$$\textcircled{4} \quad X = A BC + D + E$$

$$\textcircled{5} \quad Y = (A + B) C$$



- Q) Find the complement of the following Boolean functions.

$$\textcircled{4} \quad X = A(B+C) \quad \textcircled{5} \quad (A+B)(C+D)$$

Sol: $\textcircled{4} \quad \overline{X} = \overline{A(B+C)}$ $\textcircled{5} \quad \text{Let } Y = (A+B)(C+D)$

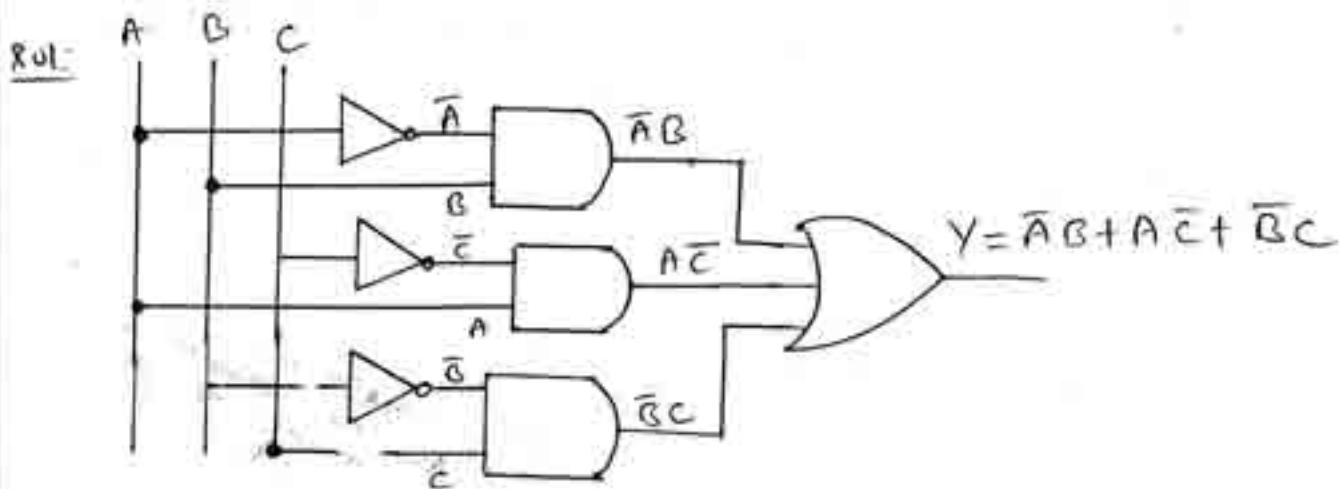
$$\Rightarrow \overline{X} = \overline{A} + \overline{B+C} \quad \Rightarrow \overline{Y} = \overline{(A+B)(C+D)}$$

$$= \overline{A} + \overline{B} \cdot \overline{C}$$

$$\overline{Y} = \overline{A+B} + \overline{C+D}$$

$$= \overline{A} \cdot \overline{B} + \overline{C} \cdot \overline{D}$$

10) Realize the Boolean expression $Y = \bar{A}B + A\bar{C} + \bar{B}C$



11) Implement the function using the truth table shown below, using minimum number of gates.

INPUT			OUTPUT
A	B	C	y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Ans: From the truth table, the output Y is 1 when

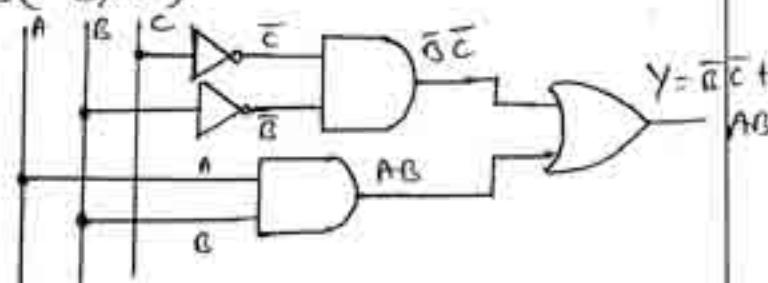
$A = 0, B = 0, C = 0$, $A = 1, B = 0, C = 0$, $A = B = 1, C = 0$ &

$A = B = C = 1$

$$Y = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

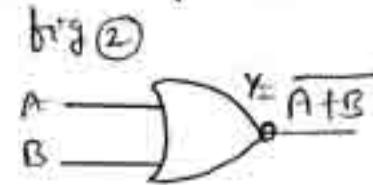
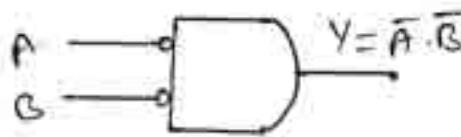
$$= (\cancel{\bar{A}} + \bar{A}) \bar{B}\bar{C} + ABC(\cancel{\bar{C}} + \bar{C})$$

$$= \bar{B}\bar{C} + AB$$



(2) Show that the bubbled AND gate is same as NOR gate:

Rul: Bubbled AND is shown in fig(1) | NOR gate is shown in fig(2)



Fig(1)

$$\text{Output } Y = \bar{A} \cdot \bar{B} // -①$$

From ① & ②, we can conclude that the bubbled AND gate is same as NOR gate

Output

$$Y = \bar{A} + \bar{B}$$

$$Y = \bar{A} \cdot \bar{B} // -②$$

(∴ From De Morgan's Law
 $\bar{A} + \bar{B} = \bar{A} \cdot \bar{B}$)

(3) Write the Dual of following expression

$$④ A + 1 = 1 \quad ⑤ A \cdot 1 = A \quad ⑥ A + BC = (A+B) + (A+C)$$

Rul:

$$④ \text{ Given } A+1=1$$

Dual of $A+1=1$ is

$$A \cdot 0 = 0 //$$

$$⑤ \text{ Given } A \cdot 1 = A$$

Dual of $A \cdot 1 = A$ is

$$A + 0 = A //$$

$$⑥ \text{ Given }$$

$$A+BC = (A+B) + (A+C)$$

Dual of $A+BC = (A+B) + (A+C)$ is

$$A(B+C) = (AB)AC //$$

Hint: ① Replace AND to OR & OR to AND
 $(\cdot \text{ to } +)$ $(+ \text{ to } \cdot)$

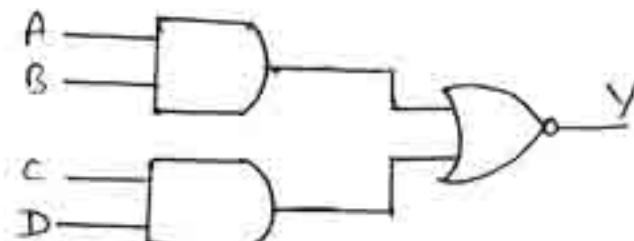
② Replace 1 to 0 & 0 to 1

Draw

~~AND-OR-Invert (AOI)~~ AND-OR-Invert (AOI) circuit & write the output expression

Rul:

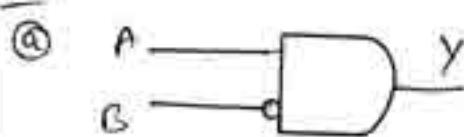
$$Y = \overline{AB+CD}$$



(15) Explain inhibit & enable operation for

① Two input AND gate ② Two input AND gate with enable

Rul.



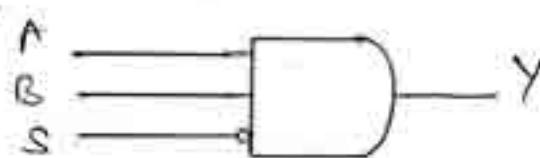
Output is,

$$Y = A \bar{B}$$

If $B=0$, $Y=1$ (Enable)
if $A=1$

If $S=1$, $Y=0$, (Inhibit)

⑥

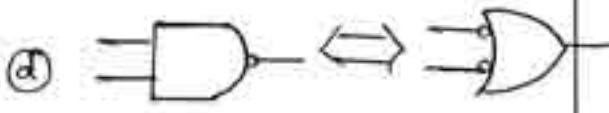
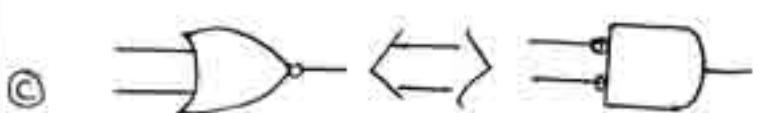
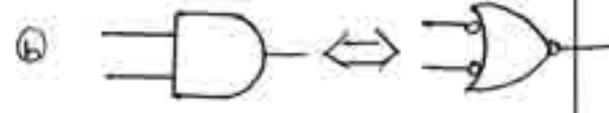
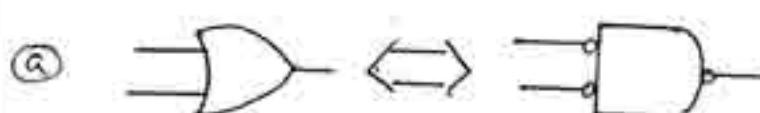


$$\text{Output, } Y = A B \bar{S}$$

If $A=1, B=1, S=0$, then $Y=1$,
then gate is enabled

If $A=1 @ 0, B=1 @ 0, S=1$,
the output $Y=0$, then the gate
is inhibited

(16) Prove the following equalities.



Rul: Let A, B be inputs, Y be output

① $Y = A+B = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A+B$

② $Y = A \cdot B = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$

③ $Y = \overline{A+B} = \overline{\overline{A} \cdot \overline{B}}$

④ $Y = \overline{A \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}}$

DeMorgan's theorem

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

(17) SIMPLIFY the following

① $XY + X'Z + YZ$ ② $\overline{(A+B)(A+C+\overline{D})(A+\overline{C}+D)(B+\overline{C})}$

③ $\overline{\overline{A}(B+\overline{C})(A+\overline{B}+C)(\overline{A}\overline{B}\overline{C})}$ ④ $\overline{(B+\overline{C})(\overline{B}+C)(\overline{A}+B+\overline{C})}$

RUL

$$\begin{aligned}
 \textcircled{a} \quad \text{Let } A &= XY + X'Z + YZ \quad (\text{Here } X' = \bar{X}) \\
 &= XY + X'Z + YZ(X+X') \quad (\because X+X'=1) \\
 &= \underline{XY} + X'Z + \underline{XYZ} + X'YZ \\
 &= XY(1+\cancel{Z}) + X'Z(1+\cancel{Y}) \\
 &= XY + X'Z
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{b} \quad \text{Let } X &= \overline{A(B+C)} \cdot (A+\bar{B}+C) \cdot \overline{\bar{A}\bar{B}\bar{C}} \\
 &= (\bar{A}) + (\bar{B} + \bar{C}) (A + \bar{B} + C) (\bar{A} + \bar{B} + \bar{C}) \\
 &= (A + \bar{B} \cdot \bar{C}) (A + \bar{B} + C) (A + B + C) \\
 &= (A + \bar{B}C) (A + \bar{B} + C) (A + B + C) \\
 &= (AA + A\bar{B} + AC + A\bar{B}C + \bar{B}\bar{B}C + \bar{B}CC) (A + B + C) \\
 &= (A + A\bar{B} + AC + A\bar{B}C + \bar{B}C + \bar{B}C) (A + B + C) \\
 &= (A(1 + \bar{B} + \cancel{C} + \bar{B}C) + \bar{B}C)(A + B + C) \\
 &= (A + \bar{B}C) (A + B + C) \\
 &= AA + AB + AC + A\bar{B}C + \bar{B}\bar{B}C + \bar{B}CC \\
 &= A + AB + AC + A\bar{B}C + \bar{B}C \\
 &= A(1 + B + \cancel{C} + \bar{B}C) + \bar{B}C \\
 &= \underline{A + \bar{B}C}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{c} \quad \text{Let } Y &= (A+B)(A+C+\bar{D})(A+\bar{C}+D)(B+\bar{C}) \\
 &= (AA + AC + A\bar{D} + AB + BC + B\bar{D}) \quad [AA=A] \\
 &\quad (AB + A\bar{C} + B\bar{C} + \bar{C}\bar{C} + BD + \bar{C}D) \quad [\bar{C}\bar{C}=\bar{C}] \\
 &= (A + AC + A\bar{D} + AB + BC + B\bar{D})(AB + A\bar{C} + B\bar{C} + \bar{C} \\
 &\quad + BD + \bar{C}D)
 \end{aligned}$$

$$\begin{aligned}
 &= [A(1 + C + \overline{D} + B) + BC + B\overline{D}] [AB + \overline{C}(A + B + \overline{1} + D) \\
 &\quad + BD] \\
 &= [A + BC + B\overline{D}] [AB + \overline{C} + BD] \\
 &= AAB + A\overline{C} + ABD + AB\overline{B}C + B\overline{C}\overline{C}^{\circ} + B\overline{B}CD + \\
 &\quad AB\overline{B}\overline{D} + B\overline{C}\overline{D} + B\overline{B}\overline{D}\overline{D}^{\circ} \\
 &\quad \cancel{AB + A\overline{C} + ABD + ABC} + BCD + \cancel{AB\overline{D}} + \cancel{B\overline{C}\overline{D}} \\
 &= [AB(1 + D + \cancel{C} + \overline{D}) + A\overline{C} + BCD + B\overline{C}\overline{D}] \\
 &= AB + A\overline{C} + BCD + B\overline{C}\overline{D} \\
 &= A(B + \overline{C}) + B(CD + \overline{C}\overline{D}) //
 \end{aligned}$$

④ Let

$$\begin{aligned}
 Y &= \overline{(B + \overline{C})(\overline{B} + C)(\overline{A} + B + \overline{C})} \\
 &= \overline{(B\overline{B}^{\circ} + BC + \overline{B}\overline{C} + \cancel{C}\overline{C})(\overline{A} \cdot \overline{B} \cdot \overline{C})} \\
 &= \overline{(BC + \overline{B}\overline{C})(A\overline{B}C)} \\
 &= \overline{AB\overline{B}CC + A\overline{B}\overline{B}C\cancel{C}^{\circ}} \\
 &= \overline{0} \\
 &= // \quad \text{⑤ } Y = \overline{(B + \overline{C})(\overline{B} + C)(\overline{A} + B + \overline{C})} \\
 &= \overline{(B + \overline{C})} + \overline{(\overline{B} + C)} + \overline{(\overline{A} + B + \overline{C})} \\
 &= (\overline{B} \cdot C) + (\overline{B} \cdot \overline{C}) + (\overline{A} + B + \overline{C}) \\
 &= \overline{B}C + \overline{B}\overline{C} + \overline{A} + \overline{B} + \overline{C} \\
 &= \overline{B}C + B(\cancel{C} + \overline{1}) + \overline{A} + \overline{C} \\
 &= \overline{B}C + B + \overline{A} + \overline{C} \\
 &= B + C + \overline{A} + \overline{C} = \overline{B + \overline{A} + \overline{C} + C} \\
 &= B + \overline{A} + 1
 \end{aligned}$$